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APPLIED MECHANICS REVIEWS

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THE GROWTH AND PRESENT STATE OF THREE-DIMENSIONAL PHOTOELASTICITY

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INTRODUCTION

THE science of photoelasticity has been rather slow in developing. If we count from 1816 when David Brewster discovered the basic phenomenon of double refraction, this subject is now 136 years old and it is only in recent years that it began to approach full maturity.

Twenty-five years elapsed from Brewster to Neumann (1),[1] who in 1841 formulated the stress optic law in terms of strain, and ten more years went by before Maxwell (2) reformulated the law in terms of stresses. It took another fifty years for the first engineering applications to appear. This happened at the turn of the century when the French engineer Mesnager and the British scientists Coker and Filon became actively interested in photoelasticity.

For nearly 35 years the work in photoelasticity was limited to two-dimensional problems. It was not until 1935 that a quantitative solution of a three-dimensional problem was obtained photoelastically by Oppel (3). Since then, however, progress has been more rapid. At times developments were so fast that books on the subject, or at least chapters, became obsolete before first printings were exhausted.

It is the purpose of this paper to review briefly the growth of three-dimensional photoelasticity from the work of Oppel to the present time when the optical method of stress analysis may be viewed as a practical instrument capable of the exploration of the complete state of stress in general space problems.

THE FROZEN PATTERN

Most of the progress in three-dimensional photoelasticity made thus far is due to the development of the frozen stress pattern. These patterns are indeed remarkable and curious phenomena. On the surface, at least, they seem to violate two of the fundamental laws of mechanics. They retain stresses after the loads inducing them are removed, and what is equally puzzling, they are not affected by changes in the geometry. A model with frozen stresses can be cut into pieces without disturbing the fixed stresses in each piece.

An explanation of this amazing phenomenon is offered by modern chemists who view the plastics exhibiting the frozen patterns as diphase materials consisting of two distinct networks having different properties.

[1] Numbers in parentheses refer to the Bibliography at end of the paper.

However, the essential property responsible for the frozen pattern is the difference in the fusibilities of the two networks. Thus in Fosterite one group of bonds becomes soft and fuses at about 85 C (the critical temperature), whereas another network remains infusible and elastic for a very much higher temperature. At the critical temperature the loads are carried by the primary, infusible and elastic network. Upon cooling the model, the soft material becomes hard again. Hence, if the model is cooled and the loads are not removed until room temperature is restored, no external loads are needed to maintain the deformation in the primary elastic network.

These deformations are sustained by the hardened secondary bonds with little change in magnitude. The stresses are then said to be congealed, fixed, or frozen, and the shape of the model can be changed at will without disturbing the frozen stresses.

Hetényi (4) carried out fundamental experiments on Bakelite relating to the critical temperature and to the elastic nature of the frozen stresses.

The two networks referred to above differ also in their moduli of elasticity, ultimate strengths, and Poisson's ratios. The diphase character extends in some instances, even to the optical properties. For example, if a Fosterite disk with a frozen pattern produced by compressive forces is further compressed at room temperature, the model becomes optically unloaded, i.e., the fringe orders are reduced. In Fosterite the two networks behave like crystals of opposite signs. However, this is not true in Bakelite where the two networks behave like crystals of the same sign. This explains why in Fosterite the fringe value of the frozen pattern is somewhat lower at room temperature than it is at the critical temperature, and why in Bakelite the opposite is true.

It is interesting to trace the development of the frozen patterns. A reference to frozen stresses was made by Maxwell (2) in 1853 who observed this phenomenon in an isinglass gell. In 1923 Filon and Harris (5) published a paper, "On the diphasic nature of glass as shown by photoelastic observations."

In 1927 Tuzi (6) made the first quantitative experiments with frozen stresses. He heated Phenolite bars to 130 C and quenched them in water at 20 C. He then removed transverse sections from the quenched bars, and from the frozen patterns which these sections exhibited he calculated stresses on the assumption of plane stress systems and normal fringe values.

In 1935 Solakian (7) published a frozen pattern from a twisted Marblette shaft. However, it seems reasonably certain that none of the above clearly understood the phenomenon and its impli-

cations. The first investigator to use the frozen pattern with complete understanding was probably Oppel (3) who in 1936 determined the stresses in a small cube due to a concentrated load.

THE STRESS OPTIC LAW AND THE SCOPE OF PHOTOELASTICITY

As already stated, the stress-optic law was originally formulated by Neumann (1) in 1841 and restated independently in 1853 by Maxwell (2). This law relates birefringence to the state of stress for a ray having an arbitrary orientation relative to the direction of the principal stresses at a point, i.e., for oblique incidence.

Recent mathematical re-examinations by Drucker and Mindlin (8) and Mindlin and Goodman (9) showed the existence of negligible rotational effects. A simplified treatment of the rotational effect is given by Jessop in a recent paper (10). The basic aspects of the law remain unchanged, however, and the stress optic law continues to be used in the form given by Maxwell. Drucker (11) utilized the rotation of the plane of vibration to study stress concentrations in thin plates with holes in transverse bending. This treatment is based on the assumption of linear bending stresses and requires verification for thick plates.

The frozen pattern provided a direct method to check the stress-optic law, and extensive work in this direction resulted in complete corroboration. An interesting pedagogical by-product of these efforts was the development of a direct photoelastic proof of the stress-optic law independent of the work of early physicists (12).

It follows from the stress-optic law that from strictly photoelastic methods, i.e., from the stress patterns and isoclinics, it is impossible to find the actual principal stresses at a point. It is only possible to find the differences between the normal stress components and the shearing stresses from which in turn the principal shears can be found. This is equivalent to saying that from strictly photoelastic data it is possible to determine Mohr's circle but not its origin.

The above limitation does not hold at free surfaces where the state of stress is similar to that in two-dimensional problems and where both principal stresses can be found. Until 1951 the applications of three-dimensional photoelasticity were, in the main, to free surface stresses and principal shears.

SURFACE STRESSES WITH BAKELITE MODELS

Among the first applications of three-dimensional photoelasticity in this country was Hetényi's (13) study in 1942 of bolt and nut fastenings. This study deals with an important and difficult problem in which the boundary conditions are not specified by forces but depend upon the deformations. The essential difficulties in this type of problem are still to be overcome.

In 1942 Frocht and Leven (14) studied the state of stress around a circular hole in a thick plate in tension. Among other things it was shown that the factors of stress concentration remain essentially equal in thick bars and in thin bars. This finding tended to correct a false impression to the opposite effect.

Interestingly enough, the conclusions from this first study made with Bakelite models were subsequently fully substantiated by the mathematical work of Sternberg and Sadowsky (15).

During 1943 and 1944 Frocht (16, 17) published studies of stress concentrations in circular shafts with transverse holes in tension and bending. Of particular interest was the study in bending which showed the possibility that the maximum stresses may occur below the extreme fibers of the shaft rather than at these fibers, a condition also suggested by some fatigue results. This paper was also of interest for other reasons. It provided a simple and direct correlation between factors of stress concentration and strength reduction. It coordinated much uncoordinated data from fatigue tests and strain measurements, and it gave a simple method for the calculation of the stresses.

In 1943 Frocht and McDowell studied the surface stresses in Diesel engine valves using small Bakelite models of the prototype. The results were not published until 1951 (18) when the problem was re-examined by Leven with relatively large Fosterite models.

FOSTERITE

A new era in three-dimensional photoelasticity began in 1947 when Leven (19) presented his paper on Fosterite. This material has two decided advantages over Bakelite. Whereas Bakelite was limited to plates 12 in. × 6 in. ×1 in., approximately, Fosterite became available in cylinders 6 in. to 8 in. in diameter and about 30 in. long, and also in flat plates about 2.5 in. thick. It thus became possible to make larger and more complex models. Fosterite was also free from annoying time stresses or edge effects.

In 1949 Leven (20) published a study of stresses in keyways in which Fosterite models were used for the first time. He removed a thin outer shell of 0.050 in. from which the surface stresses were determined optically. The shearing stresses were also checked by the numerical solution of Poisson's equation and by means of scattered light.

In 1950 Hartman and Leven (21) published the first major study of stress concentrations in three dimensions. Using large Fosterite shafts they determined the factors of stress concentration for fillets in bending for the range of r/d between 0.02 and 0.5 and D/d equal 2 and 1.25.

In 1951 the Diesel engine valves previously referred to were re-examined by Leven using large Fosterite models and the results were presented before the First U. S. National Congress of Applied Mechanics (18).

An interesting departure from the frozen pattern was made in 1949 by A. J. Durelli (22), who carried out experiments on the possible use of creep as a substitute for frozen patterns.

WORK ABROAD

From 1934 to about 1947 there was little activity in England. The famous photoelastic laboratory at University College, London, had been practically abandoned after Coker's retirement in 1934. In spite of the common language and close contact during the war years, the British seem to have had little knowledge of the development of photoelasticity in America until 1946. During that year, Jessop, who had been associated with Filon from 1919 to 1938, reinstated and modernized Coker's neglected laboratory. Since 1946 activity in Great Britian sharply increased. In 1949 Jessop (23) published an extension to three-dimensions of the Lamé-Maxwell equation, which made it possible to determine the principal stresses along a stress trajectory in a plane of symmetry. Another paper by Jessop (10), dealing with scattered light, was published in 1951. In this paper Jessop applies the method of scattered light, inaugurated by Weller (24) in 1939 and by Menges (25) in 1940, to several problems. Of particular interest is the study of stresses at a fillet in a twisted shaft.

In 1950–51 Brown and Hickson (26) working at the National Physical Laboratory published two papers showing the possibilities for greater accuracy in surface stresses by the use of sensitive photometric devices. They used sections .03 inches thick and measured birefringence with an accuracy of 0.005 fringes. Interesting industrial applications are also being made in British industry. Thus, H. Spooner of Rolls-Royce in England, exhibited some striking models of frozen turbine blades at the SESA meeting in Philadelphia last December.

In Germany, Oppel was followed by Hiltscher (27) who in 1938 brought to photoelasticity the well-established techniques

of convergent light long and efficiently used in crystallography. Kuske (28), also in 1942, made substantial improvements in these techniques. He showed that the ordinary polariscope can be easily adapted to convergent light, and that there exists no need for a special petrographic microscope.

Föppl and Mönch (29) report practical applications to pressure vessels with flanged openings. Favre and Gilg (30) studied stresses in bent plates by means of laminated models in which the layers had different optical sensitivities.

SEPARATION OF PRINCIPAL STRESSES

Prior to 1951 efforts to determine interior principal stresses were limited to special cases. One of these was the case of pure torsion. This problem was studied by Mindlin and Drucker (31) in 1938 and was solved by Frocht (32) in 1943. In 1947 Drucker and Frocht (33) published a paper showing the equivalence of scattering patterns in torsion with membrane contours for the sections.

In 1949 O'Rourke and Saenz (34) from the University of Michigan successfully determined the residual stresses in bodies having complete rotational symmetry without resorting to slicing. The problem arose from an attempt to find residual stresses in glass insulators and was treated by means of Abel's equation.

The first major attempt at the separation of the principal stresses in a general case was made by Prigorovsky and Preiss (35) in 1949 in the USSR. In an article published during that year they develop the suggestion to supplement the optical measurements from slices containing frozen stresses with mechanical measurements of deformation and strain after annealing the frozen slices. This method would seem to solve the general problem at least in theory. (It is highly doubtful whether the accuracy of such a procedure could be high.) However, closer analysis shows (36) that the method completely breaks down for values of Poisson's ratio (ν) equal to 1/2, and that it becomes highly inaccurate when ν is approximately equal to 1/2. For, when $\nu = 1/2$, there exists no one-to-one correspondence between strains and stresses, i.e., the stresses cannot be determined from the strains.

Unless, therefore, a model material is available for which ν is appreciably less than 1/2, the method breaks down. Prigorovsky and Preiss used a model material identified as IK-44 for which $\nu = 0.41$. Careful measurements have shown that the values of ν for Fosterite and Bakelite, i.e., for the materials used in this country, are nearly 0.50. For such materials the method of mechanical measurement after annealing would contribute nothing to the solution of the problem.

THE SHEAR DIFFERENCE METHOD

In 1951 Frocht and Guernsey (36) extended the shear-difference method (37), which had long proved its effectiveness in plane problems, to the general problem in three dimensions. In a paper before the First U. S. National Congress of Applied Mechanics a theory was developed showing how six rectangular stress components can be found at any point. The essential elements of the method are frozen stress patterns and the shear-difference method.

This method has been applied to a diametrically compressed sphere (38) for which Sternberg and Rosenthal (39) have now obtained a theoretical solution. A comparison of the results showed a remarkable degree of agreement. Thus, at the center of the sphere the experimental value for the major stress is $-2.59\ P/A$, while the theoretical value is $-2.62\ P/A$. Thus the prospect for satisfactory accuracy looks very promising, to say the least.

Moreover, the proposed method (36) is not limited to the elastic state. It is equally applicable to nonlinear stress-optic relations and therefore to the plastic state of the model. These developments complete the theoretical solution of the general three-dimensional problem.

Although ordinary precautions yielded satisfactory results, photometric methods (26) promise even better results. The weakest factor in the shear-difference method has been the isoclinics which are at times vague and difficult to locate. The use of photometric devices makes it possible to locate the centers of such isoclinics with precision and thereby removes the major source of error. The use of these devices may also alter the established technique of measuring birefringences by visual observation. It is quite probable that, in the future, deformations and loads will be kept to a minimum and retardation will be measured by compensation photometrically. With this new procedure the major source of error will stem from initial stresses.

MATERIALS

In addition to Bakelite and Fosterite, considerable work has been done in this country on Kriston (40), in the USSR on IK-44 (35), in Germany on Trolon and Dekorit (29), and in France on Ethoxyline resins (41).

The search continues for a castable material of a high figure of merit. Until such a material is found, the nearest substitute would be a good cementable material. Mr. Hui Pih, working in the Photoelastic Laboratory at Illinois Institute of Technology, has found that Castolite, which is a modified polystyrene, available in liquid form, can be effectively cemented or bonded. A preliminary report of this work is scheduled for presentation before the forthcoming Congress in Istanbul (42). Experiments show that the stress patterns from cemented Castolite models are little disturbed by the bonded joint. Leven is working at the Westinghouse Research Laboratories on methods to cement Fosterite and to improve the annealing thereof. In the writer's Laboratory, Fosterite was successfully cemented with a Castolite monomer, without introducing significant cementing stresses. In a recent paper by Jessop and Snell (43) mention is made of a cast Marco model.

SUMMARY

Three-dimensional photoelasticity today is a practical instrument of stress analysis capable of the complete exploration of stress distributions in general space problems. The method is no longer confined to surface stresses or to principal shears. Nor is it limited to sections of symmetry or to special cases. It is now possible to determine the complete state of stress, that is, the actual principal stresses at each and every point of a general space problem. The only limitations in the method at the present time are the difficulties in making the model and the difficulties in the application of the loads. If a model can be made and the loads can be properly reproduced, the stresses can be determined with a high degree of accuracy, initial stresses forming the major source of error.

The above discussion covers static stresses. As far as dynamic stresses are concerned, the only approach to three-dimensional problems would seem to lie in scattered light. It is altogether likely that considerable activity in that direction will soon become manifest.

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Communications

Correction to Rev. 1674 (June 1952):

The author's name should read Tskhadaya.

Theoretical and Experimental Methods

(See also Revs. 2282, 2285, 2330, 2332, 2347, 2348, 2349, 2408, 2409, 2412, 2423)

2238. Blaschke, W., Introduction to differential geometry [Einfuehrung in die Differentialgeometrie], Berlin, Springer-Verlag, 1950, vii + 146 pp. $5.

Book deserves particular attention of all engineers interested in differential geometry because it is one of a vanishingly small number of textbooks introducing the reader to a new and important trend in that branch of mathematical science. While the classical Monge-Gaussian differential geometry derives the geometric properties of surfaces from the two "Fundamental Forms," which are homogeneous quadratic expressions in the differentials of the surface parameters, a significant turn with great new potentialities has been successfully achieved by the outstanding contemporary exponent of the brilliant French school of geometers, E. Cartan (born 1869). His starting point is a generalization, introduced by G. Darboux (1842–1917) and A. Ribaucour (1845–1893), of Frenet's "accompanying trihedron," a generalization which reduces the study of surfaces to that of a two-parametric movement of the trihedron apex, with one leg piercing the surface normally, along the latter. In consequence of the fact that derivatives of the first order only occur in Frenet's formulas, the motion of the generalized trihedron is given by means of expressions, which are homogeneous and linear in the differentials of the two parameters; so we arrive at the linear Pfaffian (J. F. Pfaff, 1765–1825, teacher of Gauss) Forms, replacing the squares which occur in the two Fundamental Forms; the new basis of the study of surfaces is "linear" and not "quadratic"; for the significance of this circumstance see the review following. Judging from a purely geometric point of view, the linear study of surfaces is equivalent with the quadratic theory; in other words, the new basis permits

development of the geometric properties of surfaces from the Pfaffian forms in a manner analogous to that used in the Monge-Gaussian differential geometry on the basis of the quadratic forms. Such a structure and study of the theory of surfaces on the new basis and under restriction to purely geometric considerations is the subject of Blaschke's "Einfuehrung"—not to be confused with his "Vorlesungen ueber Differentialgeometrie I" [Dover Publications, New York, 1945], a book devoted to the presentation of the classical Monge-Gaussian differential geometry on the basis of the two fundamental forms. Blaschke is a well-known authority and prolific writer on geometry, and his books enjoy deserved popularity. To a reader desiring to familiarize himself to some extent with the purely geometric aspects of the Cartanian method, a study of Blaschke's "Einfuehrung" can be warmly recommended. In such a study, however, his "Vorlesungen" should be used as a helping guide. To support this suggestion by an illustrative example, reviewer mentions here only the section concerning Darboux's "Kinematic interpretation of Frenet's formulas" [pp. 28–30 of the "Vorlesungen"], which is very helpful for a thorough understanding of the fundamentally important section 21, "Accompanying trihedron" [pp. 12–14 of the "Einfuehrung"]. The latter title ("Introduction") of the book becomes clear after reading the review following.

I. Malkin, USA

2239. Finikov, S. P., Cartan's method of exterior forms in differential geometry [Metod vnyeshnikh form Kartana v differentsial'noĭ geometrii], Moscow-Leningrad, Gosud. Izdat. Techn.-Teor. Lit., 1948, 432 pp.

If curvature and torsion of an analytic curve in space are given as functions of the arc length, the curve can be determined by integration of the differential system represented by the three original formulas of Frenet, which is a system of linear ordinary differential equations (see W. Blaschke, "Vorlesungen," I, 1945, p. 37). In the case of the generalized Frenet system (see preceding review) an analogous problem of integration arises for determination of a surface, but the problem now is one in (linear) differential equations (of the first order) in two independent variables, with coefficients represented by functions of the latter—a Pfaffian system in terms of differentials of the two parameters of the surface. Of course, the problem of integration now produces much more difficult questions than in the case of the original Frenet system for a curve. However, it is still a problem of the first order only, and this is a success in the way of integration as compared with problems of the second order. A typical difficulty of integration is the following: In the case of the most general motion of a trihedron, its instantaneous position is determined by six independent functions, namely, the three components of displacement of the apex A of the trihedron and, say, the three Eulerian angles of rotation of the trihedron. If the motion of the latter's apex is to occur along an analytic surface in a manner indicated in the preceding review, then the six functions just mentioned, which now become functions of the two parameters on the surface, are no more independent of each other, and must satisfy certain conditions; this is a consequence of the fact that in the quadratic theory of surfaces the six coefficients of the two fundamental forms must satisfy the well-known condition equations of Gauss-Mainardi-Codazzi. So the question of integration of the generalized Frenet system involves the problem of compatibility of the six aforementioned functions of the two surface parameters in the first place. This is significantly illustrated by the subtitle of the book under review: "Theory of compatibility of systems of differential equations in total differentials and in partial derivatives." For the treatment of the problem of integration of the Pfaffian system with the conditions of compatibility taken into account, the necessary analytical apparatus has been designed by Cartan; it is based on the notion of his "exterior differential" (see W. Blaschke, "Einfuehrung," pp. 35–40). While Blaschke's book is confined to applications of Cartan's method to the classical problems of differential geometry from a purely geometrical point of view, Finikov's book is entirely devoted to the analytical problem of integration. Some readers might even be inclined to apply to Finikov's book a modification of a famous aphorism encountered in Lagrange's preface to his "Mécanique analytique": "On ne trouvera point de figures dans cet ouvrage" ("One won't find any geometry in Finikov's book"). This would be an unjust judgment. In the preface to his book, Finikov enumerates an impressive series of geometric applications of Cartan's method to important practical problems, e.g., to the problem of geometry of a stationary flow, and he almost enthusiastically emphasizes the great potentialities of the methods from many points of view. Reviewer believes that, while many engineers might be interested in making a first acquaintance with the fundamental ideas of Cartan, the mathematicians should pay more detailed attention to Finikov's important book.

I. Malkin, USA

2240. Tompkins, C. B., Wakelin, J. H., and Stifler, W. W. High-speed computing devices, New York, McGraw-Hill Book Co., Inc., 1950, 451 pp., diagrams, charts, tables. $6.50.

This book was first assembled in a report to the Office of Naval Research and is one of the few volumes devoted to the rapidly growing field of high-speed computing machines. It is divided into three broad sections covering the philosophy and basic elements of machine computation, computing systems, and physical components. The prime emphasis is on digital computation systems although one chapter is devoted to various kinds of differential analyzers and other analog computing systems. Each chapter has a large bibliography.

Two chapters are devoted to arithmetic systems and numerical analysis. Desk calculators, punch-card computing systems are described. Brief descriptions of the large-scale digital computing-machine projects in the United States are also given, including a statement of some of the problems which have been programmed on these machines. Several methods of dated conversion and storage systems are discussed. The advantages and disadvantages of various storage systems are quite completely summarized. The book is, in fact, almost an encyclopedia of the present methods in use in this field.

N. B. Nichols, USA

2241. Cross, H., Engineers and ivory towers, edited and arranged by R. C. Goodpasture, New York, McGraw-Hill Book Co., Inc., 1952, vii + 141 pp. $3.

"Engineers and ivory towers" is strong writing for strong readers. In it are collected the best essays on the relationship of education to engineering written or spoken over a quarter of a century by Professor Hardy Cross. Although edited for consistency, there is still enough of the pungent style of the author's original speeches to provide entertainment as well as provocation for argument. Hardy Cross is seldom commonplace, never trite, and when at his best his thoughts are highly stimulating. Perhaps the book raises more questions about engineering and education that it provides answers for. Cross is still inclined to follow his own belief that "the net that catches mental fish is made of questions bearing on the subject studied."

L. E. Grinter, USA

2242. Hume, K. J., Engineering metrology, London, MacDonald & Co., Ltd., 1950, xvii + 293 pp., 196 illus. 18s.

A survey of contemporary methods and equipment for precision measurements. Chapter headings indicate the topics covered: Standards of measurement, Standardization, Manufacture and design of gauges and instruments, Optical projectors and microscopes, Linear and angular measurement, Circular division, Straightness, Flatness, Squareness, Alignment testing, Screw-thread measurements, and Surface finish. The text is written clearly and assumes only a cursory knowledge of shop terminology and practice. The instruments and gages shown in illustrations are of British manufacture. The bibliography and index are ample, and the book should be a reference of value to engineers concerned with operations involving precision measurements and inspection of precision parts.

Donald M. Vestal, Jr., USA

2243. Zanobetti, D., High speed electronic analog computers (in Italian), *Energia elett.* **28**, 12, 697–713, Dec. 1951.

Author gives first a systematic survey of the different kinds of analog computers, then circuits for addiator, differentiator, integrator, etc. The elements are combined for the investigation of linear systems, governed by differential equations with constant coefficients. Circuits for multiplication and division, the latter by use of feedback, and their application to nonlinear differential equations of servosimulators, etc., are described. Author shows how to construct, by aid of diodes, analog devices for representation of nonlinear mechanical processes. For the elements of an analog computer constructed at the University of Bologna, detailed wiring diagrams are given; the computer is similar to that at the University of Wisconsin. Finally, author discusses the selection of the units of measure, the input of initial values, and the probable errors.

F. A. Willers, Germany

2244. Leutert, W., On the convergence of unstable approximate solutions of the heat equation to the exact solution, *J. Math. Phys.* **30**, 4, 245–251, Jan. 1952.

Paper deals with problem of the class treated by O'Brien, Hyman, and Kaplan [AMR **5**, Rev. 323]. The function $u(x, t)$ satisfies the heat equation $u_t = u_{xx}$ for $t > 0$ and $0 < x < 1$, the boundary conditions $u(0, t) = u(1, t) = 0$ for $t \geq 0$, and the initial conditions $u(x, 0) = f(x)$ for $0 < x < 1$, where $f(x)$ is sectionally continuous and one-sided differentiable in $(0, 1)$. The solutions of the corresponding difference equation are said to be stable (unstable) if numerical errors do not grow (grow) during the stepping-ahead process of solution, whereas they are called convergent (divergent) if they tend (do not tend) with decreasing mesh size and fixed mesh ratio $r = \Delta t/(\Delta x)^2$ to the solution of the differential equation. The specific difference equation

$$\Delta_t v(x, t) = r\delta_x{}^2 v(x, t)$$

is shown by O'Brien (above) to be unstable for $r > {}^1/_2$. Author shows that it admits, however, for each $r > 0$, infinitely many exact solutions v_{pM}, with $M = 1/\Delta x$, which converge to the exact solution of the differential equation. That there are at all so many solutions results from the fact that these solutions are, in general, not supposed to represent $f(x)$ for $t = 0$. If a_n, b_n are Fourier coefficients of $f(x)$ and $v_{pM}(x, 0)$, respectively, then author requires that to any preassigned $\epsilon > 0$, $|b_n - a_n| < \epsilon$ for any $M > M_1$ uniformly for $n = 1, \ldots, k$, where $k \to \infty$ when $M \to \infty$. This holds apparently when $b_n = a_n$ is chosen. In the special case $f(x) \equiv 1$, however, also the more obvious requirement $v_{pM}(x, 0) = f(x)$ on the mesh points, is shown to be compatible if only $M_1 > 4(\pi\epsilon)^{-1}$, at least for $r \leqq {}^1/_4$. O'Brien (see above) assumed that in this case for all $r < {}^1/_2$ the solution converges.

A. van Wijngaarden, Holland

2245. Gere, J. M., Charts determine stresses in truss members for H-S loading, *Civ. Engng., N. Y.* **21**, 12, 52–53, Dec. 1951.

2246. National Bureau of Standards, Computation Laboratory, Tables of the exponential function e^x, U. S. Dept. Commerce, June 1951, x + 537 pp.

Exponential function is of such fundamental importance in applied problems that this volume is a *must* for computational work. That a third edition appears in about a decade attests to its popularity and usefulness. Briefly, volume gives ascending exponential to $x = 10$ to at least 12D at intervals never exceeding 0.01. The descending exponential is tabulated to 18D at intervals of 0.0001 to $x = 2.5$. Other tables are also given. Present edition corrects one error and several misprints found in earlier editions. Values of e and its reciprocal to 2556D have been added.

Y. Luke, USA

2247. Powell, E. O., A table of the generalized Riemann zeta function in a particular case, *Quart. J. Mech. appl. Math.* 5, part 1, 116–123, Mar. 1952.

A ten-decimal table of the generalized zeta function (s,a) is presented, with $s = {}^1/_2$ and $a = 1.00(0.01)2.00(0.02)5.00(0.05)$-10.00. Modified second central differences are provided.

From author's summary

2248. Michlin, S. G., Direct methods in mathematical physics [Pryamíe metodí v matematicheskoĭ fizike], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1950, 428 pp.

The book explains the approximate solution of problems with differential and integral equations by reducing them to systems of algebraic equations.

The first of five principal parts is devoted to fundamental facts about Hilbert's spaces as far as necessary for understanding the following approximate methods. The second part deals with the energy and Ritz methods, and the third has for its subject Galerkin's way of solving problems of mathematical physics. The two last sections explain the method of least squares and of finite differences.

Book summarizes important facts which up to the present time have, for the most part, been scattered in various periodicals. It is based on rigorous mathematical grounds (abstract spaces, Lebesgue's integrals, theory of linear operators, etc.) and investigates in each special case the degree and manner of approximation reached by the method under consideration.

Theoretical results are applied to many problems occurring in various branches of mathematical physics and engineering: Theory of vibrations and waves, elasticity, potentials, etc. The book enriches technical literature and must be recommended to engineers and physicists.

V. Vodička, Czechoslovakia

2249. Tichonov, A. N., and Samarskiĭ, A. A., The equations of mathematical physics [Uravneniya matematicheskoĭ fiziki], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1951, 659 pp.

In a clear and vivid presentation, book deals with most of the fundamental methods and questions of mathematical physics.

It contains seven chapters and an appendix; each chapter includes a theoretical part, problems to be solved, and a section with applications to various questions of mathematical physics and techniques. In this way, solution is given of a large number of interesting problems on vibrations and wave theory, hydro-

and aeromechanics. Attention is paid also to the theory of potentials, heat conduction, and electricity.

Reviewer calls attention to fine sections on some special and more difficult questions, such as careful treatment of sources and sinks, the use of Green's functions, heat conduction in regions with boundaries moving with the time, discontinuities of the boundary conditions, etc.

Appendix contains fundamental facts on cylindrical and spherical functions, on Hermite's and Laguerre's polynominals; also, some tables for numerical computations are given.

V. Vodička, Czechoslovakia

2250. **Kreĭn, M. G., On some maximum and minimum problems for characteristic numbers and on Lyapunov's stability zones** (in Russian), *Prikl. Mat. Mekh.* **15**, 3, 323–348, May–June 1951.

The squares of frequencies of string vibration (λ) are given by characteristic numbers of wave equation. The extreme values of $\lambda_n(\rho)$, where $n = 1, 2, \ldots$, can be determined by finding functions $\rho(x)$ which satisfy conditions $h \leqslant \rho(x) \leqslant H$, $\int \rho(x)dx = M$. If $h = 0$, the functions belong to the class of functions $E(M, H, l)$. The minimum is achieved with characteristic functions $\rho(x) = \rho_n{}^{\mu}(x)$, where $\mu = 4\chi/Md$, and χ represents the least positive root of transcendental equation $(\chi)^{1/2}\operatorname{tg}(\chi)^{1/2} = t/(1-t)$; $d = M/H$; $t = d/l$. For $n \geqslant 2$, the maximum is obtained with infinite number of functions $\rho(x) \epsilon E$.

Assuming new conditions, author generalizes the results. For $0 < h < M/l$, one gets the inequality $(4n^2/hl^2)\chi(hl/M) \leqslant \lambda_n \leqslant n^2\pi^2/hl^2$. The upper limit is attained with a string of unit mass h and complementary mass $(M - hl)$ distributed at both ends of the string. The lower limit is reached with a string of unit mass h, divided into n equal parts and loaded with n equal masses $(M - hl)/n$ acting on the middle of each. Author extends generalization to partial differential equations and to approximative solution of $\lambda_1(\rho)$, taking into consideration the moment of inertia of string.

Borg's generalization of Lyapunov's criterion of boundedness of solutions of the linear differential equation with periodic member [*Amer. J. Math.*, 1949] is extended to new conditions for periodic function $p(x)$, which Yakubovich has obtained in another way [*Dokladí Akad. Nauk SSSR* (*N.S.*) **24**, 5, 1950].

Danilo Rašković, Yugoslavia

2251. **Hammersley, J. M., On a certain type of integral associated with circular cylinders,** *Proc. roy. Soc. Lond.* (A) **210**, 1100, 98–110, Dec. 1951.

Author discusses the numerical evaluation of a sixfold integral whose integrand is any given function of the distance between a pair of points in a right circular cylinder, the integration being over all such pairs of points. Using an analytic reduction which is independent of the form of the integrand, this integral is expressed as a single integral. The function resulting from this single integration is tabulated and the tables thus obtained extend the existing tabulation of the hypergeometric function.

From author's summary by E. J. Scott, USA

2252. **Ditkin, V. A., and Kuznetsov, P. I., Handbook of operational calculus** [**Spravochnik po operatsionnamu ischisleniyu**], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1951, 255 pp.

The book gives an account of the theory and use of the Laplace integral transformation in applied mathematics. It contains two main sections: theoretical grounds and tables of Laplace transforms.

Theoretical part defines fundamental notions and, in a rigorous presentation based on Lebesgue integrals and functions of a complex variable, it explains theorems referring to the Laplace transformation. Special paragraphs illustrate the application of this method to solving problems with ordinary and partial differential equations. A careful treatment of the generalized equation of heat conduction and of vibrations is given.

The second part contains tables of functions and their Laplace transforms. It certainly is one of the most complete collections of this kind. Altogether, 1354 formulas of different branches of applied mathematics and engineering science are introduced. The catalog includes not only usual functions, but also special higher transcendents such as cylindrical, spherical, and elliptic functions, theta and Mathieu functions, etc.

This handbook should be, in reviewer's opinion, a part of each engineer's and physicist's personal library. It also will render valuable service to mathematicians.

V. Vodička, Czechoslovakia

2253. **Adem, J., and Moshinsky, M., On matrix boundary value problems,** *Quart. appl. Math.* **9**, 4, 424–431, Jan. 1952.

In problems of vibration of continuous media, heat flow, etc., the state of the system is usually described in terms of a single function of position and time which leads to boundary-value problems of great difficulty. This note shows, with the aid of two examples, how a continuous medium can be divided into several regions, and with each region a function can be associated which describes its state. These functions are then grouped together to form a column matrix or vector which will represent the state of the whole system. The problem, therefore, reduces to a matrix boundary-value problem which, in general, is much simpler than the one dealt with in the usual formulation.

The examples discussed are (1) the flow of heat in a cross, (2) the vibration of two circular plates with an intermediate elastic medium.

James E. Martin, USA

2254. **Siegel, K. M., Three-dimensional conformal transformations,** *J. aero. Sci.* **19**, 4, 281–282, Apr. 1952.

Author proves that the three-dimensional method of inversion, valid in electrostatics, is not valid in hydrodynamics. Ed.

2255. **Mitrović, D., Huron, R., and Tomovic, R., On a new principle of construction of computing machines for solving of systems of linear equations by electric analogy** (in French), *C. R. Acad. Sci. Paris* **234**, 6, 589–591, Feb. 1952.

Condensed report on a circuit for representing the residues of the linear equations for a trial solution by voltages. The coefficients are represented by potentiometers. The residues are made equal to zero by altering the trial solution.

Methods like these have been used many years. [See W. Meyer zur Capellen, "Mathematische Instrumente," Leipzig, Akademische Verlagsgesellschaft, 1941, pp. 115–123, and I. S. Bruk, "An electric minimizator," *Dokladí Akad. Nauk SSSR* (*N.S.*) **62**, 481–484, 1948.] Hans Bückner, Germany

2256. **Mähly, H. J., Methods for the calculation of eigenvalues for elastic vibrations of anisotropic bodies** (in German), *Ergebn. Exakt. Naturwissen.;* ETH Thesis no. 1605; **24**, 402–442, 1951.

Author extends the known methods for approximating eigenvalues in problems on vibrations to cases of anisotropic and nonhomogeneous solids. He also gives some new formulas for numerical calculations.

Reviewer regards the paper as a fine example of research work. A systematic application of vector and tensor symbolics leads to a clear and elegant treatment of complicated questions.

Article is based on Hooke's law and presupposes stability of the undeformed state of the solid. Two types of boundary conditions are considered, namely, the case where the surface of the body is free of all forces, and the case when the surface does not move.

Paper contains three main parts, the first of which provides a short review of underlying equations of elasticity. It also gives a summary of fundamental properties of the characteristic functions and eigenvalues. The next part deals first of all with known methods for approximating eigenvalues. Then come obviously original §§8 and 9 devoted to computation of lower and upper limits of higher characteristic numbers. Here the important theorems of Weinstein and Temple are generalized. The third main part treats difficult questions on calculus of disturbances (Störungsrechnung); e.g., §13 is of research value showing the possibility of finding the third approximation of an (disturbed) eigenvalue from the first approximation of the corresponding characteristic function.

From a mathematical standpoint the paper is rigorous, and, of course, rather difficult. It isn't pure theory; it evidently presents methods applied and verified by author on real problems. Reviewer recommends it as an enrichment of inquiry not only to engineers and physicists, but to mathematicians as well.

V. Vodička, Czechoslovakia

2257. Fuks, B. A., and Levin, V. I., Functions of a complex variable and their applications [Funktsii kompleksnogo peremennogo i ikh prilozheniya], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1951, 307 pp.

The book is intended for engineers and physicists. It contains, in five main parts, special topics on the theory of functions in regard to technical use.

The first part deals with some facts on algebraic functions of a complex variable. From the technical point of view, considerations about singularities and expansions in series of such functions are of importance.

In the next section, authors discuss some problems about the analytical theory of differential equations. Engineers certainly will find very useful paragraphs on Fuchs' method of integrating equations by series.

The third chapter represents a fine treatment on the Laplace transformation and its inversion. Examples illustrate the concept abundantly.

The following main part deals with asymptotic expansions and their connection with contour integrals. This topic is very important and difficult to assimilate.

In the last section problems of importance in the theory of stability of vibrations (polynomials of the class H, Hurwitz's criterion, etc.) are discussed.

Reviewer recommends the book to engineers and physicists even though some of its sections seem to be written somewhat hurriedly.

V. Vodička, Czechoslovakia

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 2282, 2337)

2258. Faxén, O. H., Mechanics, Dynamics II (in Swedish), Stockholm, AB Seelig & Co., June 1952, xii + 332 pp. Sw. kr. 30.

This second part of a textbook for the Royal Institute of Technology in Stockholm contains the mechanics of bodies. It can be read independently of the first part, which contains the mechanics of particles.

Contents: Basic notions and laws of motion, moments of inertia, rachet, rigid bodies, rolling, small oscillations, forces due to rotation, relative motion, impact, Lagrange's equation, Hamilton's principle, principle of virtual work. Most of the numerous examples are new.

Some students have asked for a textbook developing the mechanics from the least possible number of basic propositions (axioms). This book is founded on the principles of linear and of angular momentum together with the mathematics needed. All theorems are proved from that starting point, e.g., the laws of action and reaction and of the kinetic energy, Lagrangian equation, etc. Attention is paid to the theory of solids to meet a difficulty, usually overlooked, due to the thermal agitation and to the fact that the smallest particles do not obey classical mechanics. Besides, there are suggestions on how to apply this axiomatic theory to practical problems. The advantage of this method is that the axiomatic part is independent of different opinions on how to define and measure basic quantities, e.g., mass and force.

O. H. Faxén, Sweden

2259. Maxwell, J. C., with notes and appendices by **Sir Joseph Larmor, Matter and motion,** New York, Dover Publications, Inc., 1952, 163 pp. $1.25.

First published in 1877, this book is an introduction to kinematics and kinetics by a great teacher. The first 90 pages could well be required reading for any first course in mechanics today. Chapters on the pendulum and on universal gravitation follow. The chapter, "On the equations of motion of a connected system," from vol. II of "Electricity and magnetism," is added in the present edition. One appendix by Larmor treats of the principle of relativity of motion; a second "aims at development of the wider aspects of the principle of least action."

Reviewer was interested in teacher Maxwell's comparison of the principle of action and reaction to the commercial transaction of buying and selling; similarly, energy and work are compared to a system of credit. Definitions of force, impulse, momentum, and acceleration are models of didactic clarity. Maxwell shows a historical feeling too often lacking in more recent writers for the scientific tyro.

A. D. Topping, USA

2260. Goldberg, M., Rotors in spherical polygons, *J. Math. Phys.* **30**, 4, 235–244, Jan. 1952.

A rotor is a convex curve which may be rotated in a polygon through every orientation while keeping contact with each side. Author considers rotors on a sphere. Kinematic construction of a rotor in a regular spherical n-gon, first, for $n = 4, 5, 6$, then for arbitrary n. Principal dimensions of the rotor are calculated.

O. Bottema, Holland

2261. Moszyński, W., On the problem of anisotropic and roll-sliding friction between solid bodies (in Polish), *Przegl. mech.* **11**, 2, 46–53, Feb. 1952.

Developing the idea of his previous paper on friction between isotropic and anisotropic bodies [title source, **10**, 1–3, 1951], author presents the relations between directions of reciprocal sliding of anisotropic bodies and acting force, least amount of work being produced. The solution employs curve of longitudinal friction $\mu_w(\delta)$ (measured in the direction of reciprocal sliding) introduced by the author, different from the curve of friction in rest $\mu(\varphi)$ (measured in the direction of acting load). Conditions of stable and unstable equilibrium of reciprocal sliding of anisotropic bodies are discussed, and their relations with shape and curvature of Moszyński's line $\mu_w(\delta)$ are presented. As an example of artificial anisotropy, longitudinal sliding of a rolling cylinder, caused by a small longitudinal force considerably smaller than the force of pure sliding friction, is considered. The general

form of lines $\mu_w(\delta)$ in this case is given and simple rules for experimental determination of these lines are presented.

J. Nowiński, Poland

2262. **Haringx, J. A., Miraculous top** (in Dutch), *Ingenieur* 64, 4, 0.13–0.17, Jan. 1952.

An analytical investigation about the Tippe top (Toupie magique). A complete treatment being very complicated, author does not consider the motion as a function of the time but confines himself to an analysis of a series of regular precessions; he is able to show why and under what conditions the top exhibits its typical behavior. The limits of the ratio of its principal moments of inertia with respect to the eccentricity of its center of gravity are determined. Author calculates the initial rotation which is needed as a minimum. His results have been confirmed by an experiment using a stroboscope.

O. Bottema, Holland

2263. **Zahorski, A., Remarks on dynamic loads in landing,** *J. aero. Sci.* **19**, 4, 258–264, Apr. 1952.

Paper discusses the dynamic stresses in a beam subject to a transient forcing function. It is shown by examples that the inertia forces are not linearly dependent on the maximum impact force and are very sensitive to the shape of the force-time curve. Author states that inasmuch as accelerations due to dynamic responses are not linear functions of landing load factors, it is much more realistic to use for all design purposes only those forcing functions that actually can occur in landing and to apply the desired safety factor to the dynamic accelerations only after the calculations are computed.

Author shows that, by applying the principle of superposition, the response to any forcing function that can be approximated by a trapezoid can be easily calculated. The number of effective modes to be taken in the determination of the acceleration is contended to be rationally given by the convergence of the *deflection* at the wing tip. This last point seems obscure to the reviewer.

Y. C. Fung, USA

2264. **Seely, F. B., and Ensign, N. E., Analytical mechanics for engineers, 4th ed.,** New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1952, xiv + 443 pp. $5.50.

Principal changes in this fourth edition of the textbook include some rearrangement of topics, addition of new problems, a slight broadening of viewpoint pertaining to coplanar force systems, elimination of some material on governors and gyroscopes. Present edition is clearly written and reviewer believes it still to be among the best of elementary books in this field. Indications of growing appreciation of virtual work as a tool in routine problems lead reviewer to remark that an introduction to this topic could well be included in a textbook at this level.

John E. Goldberg, USA

2265. **Ziegler, H., Mechanics. Vol. III Dynamics of systems (Mechanik. Bd. III Dynamik der Systeme),** Basel, Verlag Birkhäuser, 1952, 369 pp. Sw. frs. 46.80.

This book is intended to serve as a textbook of higher mechanics. The subject is presented in vector and tensor notation in an expert and systematic manner.

In a comparatively succinct treatment the author has managed in a competent manner to present a vast subject on a scientifically high level.

Book consists of three chapters. The first chapter (80 pages), "Systems with a finite number of degrees of freedom," deals with basic notion of degrees of freedom and generalized coordinates, d'Alembert's principle, the principle of virtual work, the principle of linear and angular momentum, the principle of conservation of energy, Lagrange's equations, natural and forced vibrations with a harmonic and a periodic disturbing force.

The second chapter (124 pages), "Vibrations," discusses two and more degrees of freedom, the principle of orthogonality, normal modes, Rayleigh and Ritz methods, nonconservative systems, lateral, torsional, and longitudinal vibrations of heavy beams, shafts, and strings.

The third chapter (175 pages), "Continuous matter," takes up in vector and tensor notation the equations for stress, strain, and displacement of rigid and elastic bodies, crystals and fluids, and some problems on torsion and bending. Next are considered two-dimensional stress systems, transverse and longitudinal wave propagation, conditions of plasticity, the basic equations of equilibrium and motion with and without frictional forces, potential stream fields, and some hydrodynamic problems.

The appendix (10 pages) contains principles of tensor calculus. Author has presented a subject of wide scope in a compact treatment by using vector and tensor notations; he has made the material accessible and has placed it within easy grasp of the reader by giving the derivations of the classical theory; but he has illustrated the theory with only a few example problems and has used applications rather sparingly.

It is an authoritative treatise on higher mechanics in a very instructive and original manner and it will be well appreciated by students for its clarity, conciseness, and accuracy.

Wilhelm Ornstein, USA

2266. **Charlton, T. M., A note on the effect of transient forces,** *Engineer* **193**, 5014, 300–302, Feb. 1952.

Calculation of the transient response of a simple mass-spring system to an arbitrary (nonanalytical) forcing function. The three well-known methods illustrated make use of force steps, impulses (Duhamel's integral) and half-range Fourier series. These classical methods have been superceded by operational calculus. Nothing new.

E. G. Fischer, USA

2267. **Kiper, G., Synthesis of plane link mechanisms** (in German), *VDI-Forschungsheft* **433**, 36 pp., 1952.

Paper presents a large number of solutions, many original and basic, of problems in position synthesis. Most of these are based on special geometric relations resulting (1) if the center curve (Mittelpunktkurve) degenerates into a circle and a straight line, or (2) if the prescribed conditions involve coincident positions of driver or follower.

After a summary of the basic Burmester theory of linkage synthesis, author gives four- and six-link mechanism solutions for up to nine prescribed positions of a link *point*. He then discusses problems in which corresponding *link* positions are prescribed. Methods leading to mechanisms with four, six, and more links are given. Numerous examples are worked out and explained in detail. A complete bibliography is included.

Paper is recommended to students of synthesis and to engineers seeking new linkage-design methods.

G. A. Nothmann, USA

2268. **Jania, Z., Calculating mongrel gear centers using properties of circles and hyperbolas,** *Tool Engr.*, pp. 44–45, 48, Dec. 1951.

Problem of finding center location and pitch diameter of a gear to mate with three given gears is considered. If the three gears are of equal diameter, center of fourth gear lies at intersection of two straight lines. If only two gears are of equal diameter, center of fourth gear lies at intersection of straight line and

hyperbola. The most general case of three unequal-diameter gears is not treated in detail, but it is pointed out that the center lies at the intersection of two hyperbolas.

W. O. Richmond, Canada

2269. Beyer, R., On the geometry of plane motions in gear trains with two drivers (in German), *Feinwerktech.* **56**, 3, 64–66, Mar. 1952.

Methods are presented for determining gear positions and paths of points in cycle gear trains with two drivers (i.e., no stationary gear).

G. A. Nothmann, USA

2270. Haacke, W., Remarks on the stability of a physical pendulum. II (in German), *ZAMM* **31**, 11/12, 333–338, Nov./Dec. 1951.

The stability of a physical pendulum of which the point of suspension performs a periodic motion in a vertical plane is investigated. The linearized equation of motion is an inhomogeneous differential equation of Hill's type; the mean positions of the pendulum are given by the condition that the constant term in the Fourier expansion of the particular solution vanishes; these positions are stable if the corresponding homogeneous equation has only stable solutions.

The condition determining the mean positions is considered in detail if the point of suspension moves along an ellipse of which the axes are small compared with the reduced length of the pendulum; in this case the differential equation is of Mathieu's type. It is concluded that for sufficiently large values of the frequency of oscillation, the mean positions are the same as those obtained if the point of suspension moves along a straight line in the direction of the major axis of the ellipse and if the amplitude of oscillation is equal to the eccentricity of the ellipse. The mean positions in the latter case have been investigated by Klotter [title source **19**, 5, 289–296, Oct. 1939].

A. I. van de Vooren, Holland

2271. Bush, V., and Jackson, J. E., Correction of spherical error of a pendulum, *J. Franklin Inst.* **252**, 6, 463–467, Dec. 1951.

Authors point out that equation of simple pendulum, $Ml^2(d^2\theta/dt^2) = -Mgl \sin\theta$, does not yield isochronous simple harmonic motion; hence, small variations in arc result in errors in period. Correcting device consists of tension spring connecting pendulum rod with a fixed point below point of suspension. With addition of spring, equation of pendulum is $Ml^2(d^2\theta/dt^2) = -Mgl\sin\theta - T_s$, where T_s is torque supplied by spring. Expression for spring constant is derived such that right side of equation reduces to a leading term $-Mgl\,\theta$ and a second term of order θ^5. For small amplitudes, error becomes very small.

As an example, authors compute correcting spring for a seconds pendulum and show that, for maximum angular amplitude of 0.015 radians, the restoring torque associated with a 1% increase in amplitude would depart from that necessary for isochronous motion by one part in 10^9.

Walter C. Hurty, USA

2272. Minorsky, N., On a pendulum kept up by an alternating current (in French), *C. R. Acad. Sci. Paris* **233**, 728–729, 1951.

If a pendulum containing a piece of iron is placed near an alternating current in a coil, it may begin to oscillate. The differential equations of this system are

$$d[L(\theta)i]/dt + zi = E\sin\omega t,\ J\ddot{\theta} + D\dot{\theta} + C\theta = d[\tfrac{1}{2}L[\theta]i^2]/d\theta$$

where $L[\theta]$ is mildly nonlinear, and even or odd, depending on the physical arrangement. In order to find approximately a periodic solution of this system, the author approximates θ tentatively by $\theta = \theta_0 \cos \Omega t$, inserts this into the first equation, and calculates approximately the periodic solution of the resulting differential equation for i. After resubstitution of this result into the second equation, the latter is seen to be, for a particular value of Ω, of a type which was shown by the author to possess a stable periodic solution [title source, **232**, 1060–1062, 1951]. The frequency ω of the current will, in general, not be in a rational relation with Ω.

Courtesy of Mathematical Reviews W. R. Wasow, USA

2273. Lichtwitz, O., An eccentric gear mechanism, *Machinery, Lond.* **79**, 2036, 896–899, Nov. 1951.

Article derives mathematical expressions defining the motion of an eccentric gear mechanism. By means of four numerical examples, it is shown how these expressions may be applied to obtain a desired dwell, or a specified ratio of maximum to minimum velocity of driven to driving crank.

F. L. Singer, USA

2274. Jung, R., Sliding motion on the vibrating plane—a contribution to the mechanics of swing feeding (in German), *Forsch. Geb. Ing.-Wes.* (*B*) **18**, 1, 13–24, 1952.

Thorough calculations of statics and dynamics on swing-feed conveyors are made. The relations between the swinging motion of the supporting plane and the sliding motion of the goods to be transported are clearly shown by means of a dimensionless representation of all phases of motion. It is pointed out that the direction of the straight line harmonic oscillation is of special influence to the feeding speed, the input power, and the energy consumed by friction. A swing-feed conveyor driven by means of a vibration exciter (two masses which are rotating with the same angular velocity in opposite directions) is calculated. A list of references covering earlier, essential publications on swing-feed conveyors concludes the paper.

Josef Boehm, USA

Gyroscopics, Governors, Servos

2275. Boksenbom, A. S., and Hood, R., Automatic control systems satisfying certain general criterions on transient behavior, *NACA TN* 2378, 45 pp., June 1951.

Paper concerns the general problem of control synthesis. It considers the design of a control system given certain physically realizable criteria (in the form of time integrals) concerning the transient behavior of the system. Methods are developed, through the use of the calculus of variations, whereby it is possible to determine the equations of the optimum system satisfying these arbitrary criteria. The requirement of stability does not enter into the main body of the methods considered. Examples of problems in the design of turbojet—engine control systems of first order and one degree of freedom—are considered in detail. Dynamic behavior is examined through the use of trajectories in the phase plane. General results are presented for systems of second order and several degrees of freedom.

S. Z. Dushkes, USA

2276. Letov, A. M., Limit values of the smallest characteristic number of a certain class of regulated systems (in Russian), *Prikl. Mat. Mekh.* **15**, 5, 591–600, Sept.-Oct. 1951.

Author studies nonlinear control systems in the range limited by two tangents to the nonlinear characteristic at its inflection points, and assumes that the parameters of the system are so chosen that it is stable in that range. He shows that in that range there exists a minimum characteristic number (in the sense of Lyapunov). The proof of this theorem occupies the major part of the paper and cannot be abstracted. It follows Lya-

punov's theory and shows, first, that the characteristic numbers are in an interval between the maximum and the minimum real roots of an auxiliary function obtained by that theory. This leads to another problem, namely, to determine the locus of the points at which the characteristic numbers have their maxima and minima. It is shown that the former appear for two values of another auxiliary function and the latter are contained between two limits, of which the lower is reached in the case of an ideal controller and the upper depends on the choice of the parameters. An example illustrates these conclusions.

N. Minorsky, France

2277. **Golomb, M., and Usdin, E., A theory of multidimensional servo systems,** *J. Franklin Inst.* **253,** 1, 29–57, Jan. 1952.

Paper develops general performance criteria for a system consisting of n simple servomechanisms with coupled error-measuring devices. The signals from the error-measuring devices of this n-dimensional system are linear combinations of the individual errors.

The first performance criterion developed concerns error-coefficient matrixes analogous to the error coefficients for one-dimensional servos. The coupling coefficients and the error coefficients for the n-uncoupled servos comprising the multidimensional system appear as elements of these matrixes. Authors next demonstrate that it is possible to combine stable and unstable one-dimensional servos into a stable multidimensional system, and to minimize the delay time by proper selection of the system parameters including the coupling coefficients. Lastly, rms-error and integrated-square error expressions are developed. The rms error is expressed in terms of the loop impedance matrix of the system and the spectral density matrix of the inputs. The minimization problem is discussed, and authors suggest the possibility of using attenuation-phase diagrams as an approximation method.

S. Z. Dushkes, USA

2278. **Chestnut, H., and Mayer, R. W., Servomechanisms and regulating system design. Vol. I,** New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1951, xiii + 505 pp. $7.75.

This theoretical introduction to automatic control with feedback has been written primarily for electrical engineers interested in the design and application of servomechanisms.

The first six chapters provide the necessary mathematical background for material in text. They treat linear differential equations, Laplace transforms, circuit theory, and system stability.

Chapters 7 through 14 are concerned with servomechanisms, commencing with a description of various typical elements and their transfer functions. All elements considered have lumped parameters, and effect of dead time is not discussed. After defining types of control systems essentially by the number of integrations in system loop, and introducing nomenclature and symbols of AIEE Subcommittee on Symbols for Feedback Control Systems, some space is devoted to preparation of block diagrams of typical control systems. Performance of servomechanisms for a varying sinusoidal input is then treated by complex plots of loop transfer function in both direct and inverse forms. Phase margin and maximum 'M' (ratio of absolute value of output to input) criteria are developed. Modifications of complex plots to improve performance by introducing series and feedback elements are discussed in chapter 10. This material is repeated in more convenient attenuation and phase diagrams and includes use of Nichol's charts. Chapter 13 summarizes preceding chapters by considering multiple-loop servo systems in some detail. Concluding chapter describes a graphical method of obtaining transient response to a step-function input from steady-state frequency response.

Reviewer has used this text in a graduate course on servomechanisms, and as the result of one year's experience has the following comments to make:

(a) Material in first six chapters, while related to subject, is better taught in separate courses; e.g., the Laplace transform cannot be adequately treated in one chapter. (b) Problems in text are primarily electrical and, while excellent in themselves, do not include typical process-control problems. (c) Concepts of proportional, derivative, and integral control elements and their combinations are not introduced. (d) Within its scope, subject matter is well treated and text is liked by students. (e) Bibliography of book is adequate.

William A. Wolfe, Canada

Vibrations, Balancing

(See also Revs. 2346, 2463)

2279. **Kenworthy, R. W., and Jennings, V. A., The modes of vibration of a semi-flexible rod under tension,** *J. acoust. Soc. Amer.* **24,** 1, 60–61, Jan. 1952.

The modes of vibration of a rod under conditions which are intermediate between a perfectly flexible string under tension and a stiff bar without tension are calculated from existing theory. Experimental investigation shows good agreement with theory.

From authors' summary by C. J. Bernhardt, Norway

2280. **Mindlin, R. D., Forced thickness-shear and flexural vibrations of piezoelectric crystal plates,** *J. appl. Phys.* **23,** 1, 83–88, Jan. 1952.

An approximate theory is given for the vibrations of piezoelectric crystal plates in thickness-shear and flexural modes, including the interaction between the elastic and electric fields. The problem of the rectangular, AT, quartz plate is solved, and computed frequencies are compared with measurements by Sykes. Algebraic formulas are obtained relating resonant frequencies with dimensions, elastic and electric constants, and orientation of cut.

From author's summary by H. G. Hopkins, England

2281. **Ferris, H. G., The free vibrations of a gas contained within a spherical vessel,** *J. acoust. Soc. Amer.* **24,** 1, 57–60, Jan. 1952.

Numerical values of the lowest 84 natural frequencies of vibration for the spherical acoustic resonator are calculated and listed in appropriate tables. Consideration is given to the geometrical structure of the nodal surfaces associated with each mode of vibration and to the degeneracy of each natural frequency.

From author's summary by Fritz Joseph Ursell, England

2282. **Erskine Crossley, F. R., A hyperelliptic function as a non-linear oscillation,** *J. Math. Phys.* **30,** 4, 214–225, Jan. 1952.

Author considers frictionless motion of a particle along ellipse with vertical major axis. Energy equation leads to hyperelliptic integrals

$$\omega t = L(e, k, \zeta) = {}_0\int^{\zeta} \frac{1 - e^2(1 - 2k^2 \sin^2 \zeta)^2}{1 - k^2 \sin^2 \zeta}\, d\zeta$$

where e is eccentricity of ellipse and k is related to total energy of particle.

Introducing hyperelliptic amplitude $\zeta = \text{ham}\ \omega t$, solution is given in terms of $\sin \zeta = \text{sham}\ \omega t$. Integrals L are calculated by

Simpson's rule for different values of parameters; for oscillating particle, period is given by complete integral ($\zeta = \pi/2$). Phase plane representations are given with variables φ (parameter on ellipse) and $d\varphi/dt$ for different values of total energy. Closed trajectories correspond to oscillatory motion, open trajectories to rotation along ellipse. Some difficulties of calculation for $e \approx 1$ are mentioned without solution. R. Timman, Holland

2283. Mathewson, Alice W., A manual for waveform analysis, *David W. Taylor Mod. Basin Rep.* 794, 25 pp., Jan. 1952.

In analyzing vibration records, a laborious harmonic analysis may frequently be avoided by direct comparison of wave-form records obtained by combining simple harmonic functions of various frequencies, phase relations, and amplitudes. A reasonably accurate harmonic analysis of such records is essential for their interpretation. This manual presents a simple guide for the approximate analysis of such records. A group of sample curves and their sinusoidal components, which represent the most frequent forms encountered, has been plotted for the convenience of technical personnel working in the fields of vibration, sound, and electricity.

This manual is intended to act only as a supplement to a text treating the subject of the analysis of wave forms, e.g., "Wave-form analysis" by R. G. Manley, since it is very convenient to have a set of sample curves for comparative purposes.

The wave forms included are combinations of the fundamental and higher harmonics up to and including the fourth harmonic. Each combination is plotted for phase angles from 0 to 2π taken in increments of $\pi/4$. From author's summary

2284. Shimanov, S. N., On the theory of quasi-harmonic vibrations (in Russian), *Prikl. Mat. Mekh.* **16**, 2, 129–146, Mar./Apr. 1952.

Author outlines a method (which he calls the method or auxiliary systems) for determining periodic solutions of a system o linear differential equations with periodic coefficients oscillating but little around their average values. The method consists in adding to the right side of some of these equations a certain number of constants which are so determined that the system has periodic solutions for a certain set of initial conditions. These constants are thus certain functions of the latter, and it is shown that for certain initial conditions the constants can be reduced to zero. The passage through the auxiliary system permits determining the initial conditions under which the original system has periodic solutions. The major part of the paper is devoted to the proof of this proposition. Several examples illustrate the method. N. Minorsky, France

⊗2285. Gantmacher, F. R., and Kreĭn, M. G., Oscillation matrixes and kernels and small oscillations of mechanical systems [Oszillyatsionnýe matritsí i yadra i malýe kolebaniya mekhanicheskikh sistem] 2d ed., Moscow-Leningrad, Gosud. Izd. Tekh.-Teor. Lit., 1950, 359 pp. \$2.

This new edition of a text originally published in 1941 treats small oscillations of continuous elastic systems through integral equations having oscillatory kernels. Some fundamental matrix theory is also presented. The book consists of five chapters and two appendixes.

In chap. I, the known properties of matrixes and quadratic forms are introduced. In chap. II the theory of oscillation matrixes is developed. While only symmetrical matrixes are needed for small vibration, authors present a complete theory, including unsymmetrical matrixes. In chap. III a study is made of n concentrated masses in an elastic continuum. When the influence functions of a given continuum are found to be oscillatory, the vibration theorems of n concentrated masses are obtained directly from the theorems of chap. II. The authors point out that the "oscillatoriness" of the influence function is equivalent to the fact that "under the action of n concentrated forces the deflection curve changes sign not more than n-1 times."

Chap. IV develops the integral-equation theory for free and forced vibrations of an infinite number of masses on an elastic continuum. Differential equations are not used except in determining influence functions. Kellog's class of integral equations is extended. Considerable use is made of Chebíshev's functions and Markov's functions. A new approach to the general theory of integral equations develops naturally from this physical situation. The authors consider continuous beams and strings.

Chap. V is again concerned with matrix algebra, dealing with vectors defined as "Markov systems." Inverse theorems relating given spectral characteristics to oscillation matrixes and kernels are given.

Appendix I is concerned with the iterative method of calculating characteristic numbers. Appendix II is concerned with the problem of determining the distribution and values of n concentrated masses on a string to satisfy a given set of natural frequencies.

Some 56 references to original work are given at the end. Walter W. Soroka, USA

2286. Kilpatrick, Myra F., and Kilpatrick, J. E., Torsional vibrations of coupled cylinders, *J. acoust. Soc. Amer.* 22, 2, 224–230, Mar. 1950.

Problem of torsional vibration of system consisting of two identical cylinders joined axially by a third cylinder is considered. The frequency spectrum is examined in considerable detail in relation to the various parameters of the system, and a comparison of theoretical and experimental results is made. Practical application of interest is the experimental determination of modulus of rigidity of materials. R. W. Traill-Nash, Australia

Wave Motion, Impact

(See also Revs. 2506, 2521)

2287. Gerard, G., and Becker, H., Column behavior under conditions of impact, *J. aero. Sci.* 19, 1, 58–60, 65, Jan. 1952.

Authors assume stresses are elastic and show that buckling of a bar under impact loading can take place only after the compressive stress wave has propagated over a "critical" length of the bar. From this, it is concluded that a column can momentarily support a dynamic compressive stress of any magnitude which may be introduced by impact. No reference is made to the work of M. P. White and LeVan Griffis [AMR 1, Rev. 1450], D. S. Clark, P. E. Duwez, and H. E. Martens ["The propagation of plastic strain in compression," *NDRC* Report (OSRD 3886, ATI 20421), July 1944], and others referred to in these papers. These references appear to indicate that the assumption of elastic stress and the conclusion of "dynamic stress of any magnitude" may not be completely justified in this problem. White and Griffis show that the metal flows at high impact velocities and behaves like a fluid at supersonic velocities. Authors' test technique of breaking the specimen in tension to produce the impact in compression should provide possibilities of obtaining test data in this field. However, complications may arise from the changed stress-strain properties of the material; also, the maximum obtainable velocity is only a small percentage of the sonic velocity for metals. B. E. Gatewood, USA

2288. **Benson, J. M., Havens, R. F., and Woodward, D. R., Landing characteristics in waves of three dynamic models of flying boats,** *NACA TN* 2508, 41 pp., Jan. 1952.

Powered models of three different flying boats (one model with unusually long afterbody) were landed in oncoming waves of various heights and lengths. The resulting motions and accelerations were recorded to survey the effects of varying the trim at landing, the deceleration after landing, and the size of the waves.

The data for landings with normal rates of deceleration indicated that the most severe motions and accelerations were likely to occur at some period of the landing run subsequent to the initial impact. Landings made at abnormally low trims led to unusually severe bounces during the landing run. The least severe landings occurred after a stall landing when the model was rapidly decelerated at about 0.4 g in a simulation of the proposed use of braking devices. The severity of the landings increased with wave height and was at a maximum when the wave length was of the order of from one and one half to twice the over-all length of the model.

The models with afterbodies of moderate length frequently bounced clear of the water into a stalled attitude at speeds below flying speed. The model with the long afterbody had less tendency to bounce from the waves and consequently showed less severe accelerations during the landing run than the models with moderate lengths of afterbody.

From authors' summary by Ernest G. Stout, USA

2289. **Johnson, J. W., Engineering aspects of diffraction and refraction,** *Proc. Amer. Soc. civ. Engrs.* **78,** Separ. no. 122, 32 pp., Mar. 1952.

Design, construction, and operation of many coastal engineering works are considerably dependent on principles of wave refraction and diffraction. Paper develops and illustrates principles that enable estimation of wave conditions at specified points in shallow water or at shoreline. Wave characteristics can be developed from weather observations or forecasts, or from wave recorders. Phenomenon of wave refraction on shoaling bottom, together with companion effects such as littoral currents, is discussed. Refraction of waves by currents is also considered, and formulas are developed that make possible the measurement of effect under various conditions. Wave diffraction by breakwaters is an important consideration in engineering design of harbor facilities. Means of measuring and locating area in which phenomenon will occur are given for conditions of semi-infinite breakwaters and also for breakwater gaps. Appendix presents summary of basic theory of wave diffraction and illustrates computations necessary for construction of diffraction diagram.

From author's summary by J. J. Gilvarry, USA

2290. **Hughes, D. S., and Cross, J. H., Elastic wave velocities in rocks at high pressures and temperatures,** *Geophysics* **16,** 4, 577–593, Oct. 1951.

By means of the pulse technique described in a previous paper [AMR 3, Rev. 1228], authors have measured velocities of dilatational and rotational waves in various rock samples. Samples, which were cylinders 1 in. in diam and 1.5 to 3.25 in. long, were jacketed in copper or neoprene to keep the pressurizing fluid from being forced into the sample. Velocities were determined with samples under pressures up to 5000 atm and at temperatures up to 300 C. It was found that velocities increase with pressure and decrease as temperature increases. Velocities at room temperature generally increased if the sample was first subjected to the high-pressure high-temperature condition. Data available for comparison show a substantially lower velocity in situ than in the sample. Authors attribute this to the sample having been subjected to high pressure before the measurements were made, and to dispersion. The measurements show that when a sample is covered with a layer of material in which the wave velocity is somewhat less than in the sample, the transformations required to give the late pulse arrivals characteristic of this method do not occur.

E. A. Ripperger, USA

2291. **Heins, A. E., Some remarks on the coupling of two ducts,** *J. Math. Phys.* **30,** 3, 164–169, Oct. 1951.

Using a modified Green function and a Fourier transform, a solution is given for the horizontal coupling of surface waves in two ducts. The boundary conditions are that the two ducts have different values along their upper surface but the same value on their lower surface. Charles B. Officer, Jr., USA

Elasticity Theory

(See also Revs. 2256, 2342)

2292. **Föppl, A., Lectures on technical mechanics. III. Strength of materials. [Vorlesungen über technische Mechanik. III. Festigkeitslehre],** München, Verlag R. Oldenbourg, 1951, xii + 303 pp. DM 15.

The 15th edition of this volume of Föppl's classic "Vorlesungen" has been prepared by his son Otto, as have the last five preceding revisions. Accordingly, the character of the volume has changed quite markedly from the early editions. Originally a text, it has become encyclopedic. Reviewer believes its usefulness as a text has been appreciably diminished by this change.

Sections which have undergone major revisions include: Definition of stress; derivation of differential equations of equilibrium; description of states of strain accompanying simple stress states; discussion of effects of surface conditions upon fatigue strength; and the damping capacity of materials. The revised treatment of "reduced" stress and its application to combined bending and torsion of a circular shaft are based upon maximum shear theory of failure. Limitations of the theory are noted. The discussions of the strain energy in unit volume, stresses in a thick-walled cylinder under internal pressure, and the hydrodynamic analogy for torsion have also been rewritten.

Most innovations of present volume are related to a resolution of the state of stress at a point. Primary resolution is into two parts: A state of isotropic tension (or compression), and a "shear stress condition." Author points out that for isotropic materials the deformation accompanying the first part is a simple volume expansion and depends only upon the bulk modulus of the material. Deformations accompanying second part involve merely distortion without dilatation and thus depend only upon the shear modulus. Because of this separability, he denotes bulk modulus and shear modulus as "natural" elastic constants, and designates modulus of elasticity and Poisson's ratio as "derived" elastic constants. In support of the designation "natural," author cites work of Bridgman showing systematic variation of bulk modulus with atomic number. He points out that shear modulus is determined by the state of aggregation and not at the atomic level.

For other applications, Föppl uses a further resolution of the state of stress in which the shear stress condition is subdivided into a "basic shear stress condition of the first kind" and a "basic shear stress condition of the second kind." He employs this resolution in obtaining stress distribution in thick cylinder under internal pressure. It appears to reviewer that essential simplicity of the problem is concealed by the artificial resolution.

Awkwardness is all the more evident in passing from solution for cylinder with closed ends to case of open ends (no axial tension).

A further application of the resolution of the state of stress is made in a lengthy development which purports to show a basic defect in the classical theory of elasticity. Since author's writings on this topic have already been reviewed at some length [AMR **5**, Revs. 47, 363], they will not be discussed here. Reviewer does note, however, that treatment in the present work is in terms of an awkward, cryptic notation which places an unwarranted burden upon the reader.

A highly meritorious addition is the section on the practical value of considerations based upon theory of elasticity. Author points out that exact solutions of elasticity are frequently accorded high regard without any heed being paid to the assumptions which underlie the theory. In particular, he directs attention to the imperfect elasticity of actual materials and its consequences under repetitive loading. He concludes section by emphasizing that "mechanics is a branch of physics" and not applied mathematics. R. E. Newton, USA

2293. Ôkubo, H., On the two-dimensional problem of a semi-infinite elastic body compressed by an elastic plane, *Quart. J. Mech. appl. Math.* **4,** part 3, 260–270, Sept. 1951.

Author reduces problem to that of a tall elastic rectangle (Young's modulus E_1) pressing upon an elastic half-plane (Young's modulus E_2). Series solutions are introduced and manipulated rather strenuously. Determination of coefficients of the series is to be achieved by solving an infinite set of simultaneous equations. No attempt is made to justify the many operations employed, nor is any estimate given as to what the height-to-width ratio of the rectangle should be in order to employ the method with safety. Pressure distribution at the interface and the first few coefficients for the series are given for the five cases $E_1/E_2 = 0$, 0.3358, 1, 2.978, and ∞. Author states that the results in the last case (rigid press) agree with those of Sadowsky. Robert C. Meacham, USA

2294. Isihara, A., Hashitsume, N., and Tatibana, M., Statistical theory of rubber-like elasticity. IV (Two-dimensional stretching), *J. chem. Phys.* **19,** 12, 1508–1512, Dec. 1951.

Authors show that the failure of the statistical theories of rubberlike elasticity to represent two-dimensional behavior is due to the assumption of Gaussian chains in the network. Unlike the one-dimensional case, this nonlinear character of the chains becomes important in two dimensions for quite small extensions, as had previously been observed by Treloar and others. An analysis is made using one more term than that which leads to Gaussian chains, and a result is obtained which has a form suitable for representation of the two-dimensional data. The form is similar to that used by Treloar to represent his data, but is somewhat more general in that there are three instead of two adjustable parameters (elastic constants). Unfortunately, these constants are not readily obtainable from the data, as two of them are of the order of the experimental error. An analysis of the experiments of Treloar leads to an approximate evaluation of the constants for this case.

Donald G. Ivey, Canada

2295. Pailloux, H., Passage from elasticity to strength of materials (in French), *C. R. Acad. Sci. Paris* **234,** 1, 49–51, Jan. 1952.

By expanding displacements in Taylor's series whose coefficients are functions of spatial coordinates, approximate integrodifferential equations governing displacements of prismatic beam are derived. Title is suggested by agreement of lowest-order approximation with conventional Navier-Bernoulli hypothesis. Lawrence E. Goodman, USA

2296. Peterson, R. E., Design factors for stress concentration. Part 1—Notches and grooves in bending. Part 2—Notches and grooves in tension and torsion. Part 3—Fillets in bending. Part 4—Fillets in tension and torsion. Part 5—Transverse holes in bars and shafts, *Machine Design* **23,** 2, 3, 5, 6, 7; 169–173, 161–165, 159–163, 173–177, 155–159; Feb., Mar., May, June, July 1951.

These five papers contain 19 diagrams giving values of stress-concentration factor plotted for various dimensional parameters for: (1) Notches and grooves in bending; (2) notches and grooves in tension and torsion; (3) fillets in bending; (4) fillets in tension and torsion; and (5) transverse holes in bars and shafts. The diagrams are planned carefully and drawn for excellent readability.

The accompanying text, additional diagrams, and numerical examples show clearly the methods suggested for using information from the diagrams in design. Author also indicates some of the approximations and limitations involved in application to design.

These papers should be very useful to designers concerned with strength reduction that may be caused by notches.

Horace J. Grover, USA

2297. Cheng, C. M., Resistance to thermal shock, *J. Amer. Rocket Soc.* **21,** 6, 147–153, Nov. 1951.

Author investigates thermal stresses in a plate in which the temperature is assumed to vary only through the thickness. He also assumes that the plate is initially at zero temperature and the surfaces subjected to definite rates of heating. Apparently there is also a tacit assumption that the rate of heating is sufficiently low so that inertial stresses may be neglected. Accordingly, stresses may be calculated in particular for cases in which the temperature is cycled; that is, alternately raised and lowered.

The theoretical results are used for investigating NACA cycling heat tests on ceramic and ceramal test specimens 2 in. in diam and $^1/_4$ in. thick. It is concluded that these tests, called thermal shock, are not suitable to provide reliable information on the design of structures to resist the damaging effects of heat cycling.

Reviewer considers the tests to be more nearly in the category of fatigue tests in which the stresses are alternated as the temperature is cycled. The term "shock" appears inappropriate as used. It may be that it has taken hold in industrial use and cannot now be discarded. Reviewer is also of the opinion that while the analysis of the paper provided author with an exercise in elementary stress states, it was not necessary to the position of challenging the major premise underlying the NACA tests. W. H. Hoppmann, II, USA

2298. Sengupta, A. M., Some problems of elastic plates containing circular holes. I, *Bull. Calcutta math. Soc.* **43,** 1, 27–36, Mar. 1951.

Author considers an infinite plate (disk) containing two equal circular holes subjected to the action of (a) a couple of moment M (axis of moment perpendicular to plane of plate), and (b) a center of pressure, acting (in both load cases) midway between the holes.

Using solution in bipolar coordinates which in general was given by Jeffery [*Phil. Trans. roy. Soc. Lond.* (*A*), **221,** 265–293, March 1921], author develops equations for the circumferential stresses on the edges of the holes (which are radially

unstressed). By a special artifice, the series included in these equations have been made more rapidly convergent.

Numerical results are presented for the ratio of the distance between the centers of the holes to the diameter of a hole (a) amounting to cosh 0.8 = 1.337 (for acting moment M) and (b) amounting to cosh 1 = 1.543 (for acting center of pressure).

In the latter case, using Jeffery method without mentioning it, the author arbitrarily assumes the center of pressure equal to 1. In reviewer's opinion, for engineering purposes it is necessary to multiply the stress (given by author's equation) by a factor C ($C = -p_0 r_0^2$, where r_0 is the radius of any circle around the center of pressure and $-p_0$ the radially acting pressure equally distributed over the circumference of this circle).

The stress equations are only valid within the elastic stress range.

Author does not mention the practical use of the load cases considered. Some remarks to this question are to be found in Föppl, "Drang und Zwang," vol. 1, 2nd ed., München and Berlin, Verlag Oldenbourg, 1924, page 294.

Edgar Seydel, Germany

Experimental Stress Analysis

(See also Rev. 2359)

2299. **Hardrath, H. F., and Ohman, L., A study of elastic and plastic stress concentration factors due to notches and fillets in flat plates,** *NACA TN* 2566, 23 pp., Dec. 1951.

Six large 24S-T3 aluminum-alloy sheet specimens, containing various notches or fillets, were loaded in tension. Local strains were measured by SR-4 strain gages and small gage-length extensometers. From these data and stress-strain data for the sheet material, stress-concentration factors were evaluated.

With the elastic region, experimental values of stress-concentration factor were somewhat higher than values calculated by Neuber's theory and those obtained photoelastically (by Frocht).

Stress-concentration factors decreased as local strains entered the plastic region. A generalization of Stowell's relation for the plastic stress-concentration factor at a circular hole in an infinite plate [*NACA TN* 2073, 1950] agreed well with experimental results.

Horace J. Grover, USA

Ꞩ2300. **Yarnell, J., Resistance strain gauges. Their construction and use,** London, Electron. Engng., 1951, vii + 128 pp. 12s 6d.

The chapter headings include: 1. Wire-wound strain gauges; 2. Cements; 3. Bridge theory; 4. Circuits and methods; 5. Slip rings; 6. Strain-sensitive lacquer; 7 and 8. Stress and strain in two dimensions. The first chapter explains the principles, construction methods, and provides illustrations for several types of bonded-wire gages. The construction methods are for usual types of gages and would not be of practical interest in locations where gages are available at reasonable cost. The names of most of the cements discussed are quite international, but the solutions suggested for weather protection are trade names of more local significance. Bulletins available from Baldwin-Lima-Hamilton as to gage types and method of application give better information on these particular subjects, although the use of De Khotinsky and Litharge-Glycerine cements is of added interest. The bridge theory is concerned with the best choice of resistances fór greatest voltage sensitivity or for greatest current sensitivity. Equations giving the relations are derived. Circuits and methods discuss the slide wire bridge, switching of gages, temperature compensation, a-c bridge excitation, Chopper amplifiers, and general methods of recording. The chapter on slip rings should be of value to those who must measure strain on rotating elements. Strain-sensitive lacquer is mentioned only as a method of exploration so that regions of maximum strain can be determined. The last chapter considers the 3-element rosette having angles of 45° or 120° between elements. Equations and small charts are given from which the principal strains can be determined.

This book appears to have been written by one with considerable experience in some of the techniques described; for example, problems encountered in the use of slip rings, but lacking in appreciation in many other techniques in common use, particularly in America. Frequently, numerical values are given which are misleading, i.e., the normal upper limit of strain measurement is given as 0.2%, whereas 2.0% is common for many SR-4 gages in tension, with 8–10% possible in special gages. A-c bridge-exciting frequencies go to much higher values than the 500 cps mentioned without capacity balance necessary. This may be due to the smaller and lower resistance gages common in the United States. Gages were generally stated to be linear even though the gage wires were plastically strained. It should be noted that gage-wire plastic sensitivity factor is always nearly 2, and for a gage to remain linear, its elastic and plastic sensitivity factor must be the same.

Irwin Vigness, USA

2301. **Klute, C. H., Penther, C. J., and McKee, L. B., An electrical extensometer for determining elongation of elastomers,** *ASTM Bull.* no. 180, 44–49, Feb. 1952.

Authors discuss extensometer developed for use in ASTM Standard D412-49T. Specimens contribute negligible amount of energy to sensitive elements of instrument. This energy is amplified by thyratron circuits which energize motors to furnish power to operate extensometer.

Instrument is adaptable to load-elongation diagnosis and to operate below and above normal temperatures. Data obtained are as reliable as those obtained by optimum visual techniques.

Frank J. Mehringer, USA

2302. **Rocha, M., Experimental dimensioning of structures** (in French), *Ann. Inst. tech. Bât. Trav. publics* (*N.S.*) no. 235, 40 pp., Feb. 1952.

The purpose of paper is to review the fundamental experimental methods existing in our day for the dimensioning of structures by the use of models.

The first chapter establishes the principles which govern the relationships between model and prototype, both for static and dynamic equilibrium. Author considers first the conditions of similarity for prototype and model of the same material, and then analyzes the case of prototype and model of different materials; prototype in elastic equilibrium is considered as a particular case. In the further discussion, the case of prototype in plane stress and in plane strain is treated, and the equilibrium of structures is analyzed in detail.

The following chapter deals with the choice of the proper material and the adequate selection of the scale. Some details of the technique of model loading are mentioned. The next chapter is devoted to the methods and techniques which have been developed for the stress analysis, and both direct and indirect methods are considered. Reference is made to the problem of accurately measuring strains, and standard extensometers are described from the point of view of their application to the model analysis. In author's opinion, electrical-resistance gages are the most useful for this purpose. A brief reference is made to special methods such as brittle coatings and photoelasticity.

In reviewer's opinion this paper is an excellent and concise summary of the subject, and includes a good bibliography.

Cesar Augusto Sciammarella, Argentina

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Rev. 2335)

2303. **Czitary, E., Cable tramways [Seilschwebebahnen]** Wein, Springer-Verlag, 1951, vii + 390 pp. $11.80.

Very thorough treatise includes following parts and chapters: Part A, Ch. I: Description of cable tramways; Ch. II. Types of c.t.: 1. Aerial c.t.; 2. Terrestrial c.t.; Ch. III: Wire ropes: 3. General description and historical development; 4. Wire rope specifications; 5. Manufacture; 6 to 10. Various ropes, cores and lays. Part B. Types, construction, and maintenance of c.t., including following chapters: I. Statics; II. Strength of wire ropes; III. Layout, general arrangement, number of sections, capacity and speed of carriages; IV. Supports, design of portals and piers; V. Propelling cables and their mechanism, efficiency of both supporting and propelling cables, electrical safety, and long-distance records; VI. Types of carriages for freight and persons, and their mechanism; VII. Personal and discharge terminals; VIII. Protection; IX. Construction, maintenance, costs, regulations; X. Examples. Literature. Reference is made to similar book, "Die Drahtseilbahnen" by P. Stephan (4 editions from 1907 to 1929), same publishers, 288 pp. J. J. Polivka, USA

2304. **Langhaar, H. L., Torsion of curved beams of rectangular cross section,** *J. appl. Mech.* **19**, 1, 49-53, Mar. 1952.

See AMR **5**, Rev. 383.

2305. **Dobie, W. B., The torsional strength of structural members,** *Struct. Engr.* **30**, 2, 34-46, Feb. 1952.

An analytic and experimental study is made of standard structural sections subjected to torsion. The calculations are not given but it is stated that the membrane-analogy and relaxation methods were used. An investigation indicated that the accuracy obtained was of the order of two per cent. Analytic and experimental results are plotted for various sections. Good agreement is obtained in the working range. The limiting torque for a given section is determined from (a) maximum deformation, (b) limiting stress using the elastic theory, and (c) simple plastic theory.

It is demonstrated that the simple plastic theory is most suitable for determining the limiting torque for a structural cross section. Safe limits for the limiting torque are recommended for the other two conditions, should they be the predominant factors. Vincent P. Zimnoch, USA

2306. **Lattanzi, F., Application of the theory of transverse elasticity ellipse to the study of a curved rod elastically constrained at the ends** (in Italian), *Atti Accad. naz. Lincei R. C. Sci. Fis. Mat. Nat.* (8) **11**, 1/2, 45-52, July/Aug. 1951.

The method of the transverse elasticity ellipse due to C. L. Ricci ["L'ellisse di elasticità trasversale," *Reale Accad. Sci. Torino*, 1911] is extended to the study of the deflections of curved beams bent out of their planes of initial curvature and elastically constrained at the ends. Enrico Volterra, USA

2307. **Van Zandt, R. P., Beam strength of spur gears,** *SAE Quart. Trans.* **6**, 2, 252-258, Apr. 1952.

Due to the fact that the initial clearance or interference of a pair of spur gears may be closed up by the deformations of gear teeth in action, the static load-carrying capacity of this pair of gears increases considerably. Using the Lewis formula for spur gears, the author derives a method to determine the critical value of the interference which may justify whether the mentioned phenomenon occurs in the pair of gears under consideration. A method to determine the combined stiffness of gear teeth, which dominates the critical interference, is described; test results are given to demonstrate the increase of load-carrying capacity caused by using proper interference. Ling-Wen Hu, USA

2308. **Gerard, G., Note on beams and plates,** *J. aero. Sci.* **19**, 3, 207-208, Mar. 1952.

Purpose of note is to show that the classification of a structural element as a beam or a plate does not depend solely upon the geometric parameters of the element. Extension of the results of Ashwell [AMR **4**, Rev. 1069], and Houbolt and Stowell [AMR **4**, Rev. 1533] indicates that an element of a given size behaves initially as a beam and that the flexural rigidity may grow as the bending strain is increased. True plate behavior in which anticlastic curvature is completely restrained is approached only as a limiting case. From author's summary

2309. **Lansing, W., Thin-walled members in combined torsion and flexure,** *Proc. Amer. Soc. civ. Engrs.* **78**, Separ. no. 119, 19 pp., Mar. 1952.

Stress and angular deformation calculations are made for C- and Z-section beams, using fundamental theory of J. N. Goodier [*Cornell Univ., Engng. Exp. Sta. Bull.* nos. 27, 28; Dec. 1941, Jan. 1942]. Differential equation for rotation of beam cross sections is similar to that for the deflection of a tension beam-column. This "tie-rod" analogy provides approximate solutions of easily estimable accuracy. Stress and angular deflection curves are provided for two C- and one Z-section beams. C. M. Tyler, Jr., USA

Plates, Disks, Shells, Membranes

(See also Revs. 2251, 2298, 2308, 2322, 2323, 2324, 2333, 2523)

2310. **Illing, Edith, The bending of thin anisotropic plates,** *Quart. J. Mech. appl. Math.* **5**, part 1, 12-28, Mar. 1952.

A complex fourth-order differential equation is derived for the normal displacement of the middle surface of a thin orthotropic plate subjected to transverse pressure. The displacement equation is solved for the problem of a circular plate which is clamped around its edge and bent by a typical general force applied to one face. Particular cases of the general solution are used to investigate the bending of the circular plate under the action of (1) a uniform pressure and (2) a linearly varying pressure. The normal displacement of the middle surface is also determined when the applied force varies as the square of the distance from the center of the plate.

From author's summary by D. L. Holl, USA

2311. **Clark, R. A., Gilroy, T. I., and Reissner, E., Stresses and deformations of toroidal shells of elliptical cross section with applications to the problems of bending of curved tubes and of the Bourdon gage,** *J. appl. Mech.* **19**, 1, 37-48, Mar. 1952.

See AMR **5**, Rev. 646.

2312. **von Sanden, K., and Günther, K., The strength of cylindrical shells, stiffened by frames and bulkheads, under uniform external pressure on all sides,** *David W. Taylor Mod. Basin Transl.* **38**, 81 pp., Mar. 1952.

Translated from *Werft und Reederei* **1**, 8, 9, 10, 1920; **2**, 17, 1921.

2313. Grioli, G., On elastic deformations of a homogeneous shell subjected to pressure or tension (in Italian), *R. C. Semin. mat. Univ. Padova* **20,** part 2, 278–285, 1951.

Author's investigation gives some indication on stresses and change of volume in an elastic body with a hole, submitted to uniform pressure on the external boundary and on the surface of the cavity. Result is obtained after Signorini's research [see *Ann. Scu. norm Pisa* (II) **II,** 1933] and shows that stress in bodies with cavity is always greater than stress in a simply connected body, submitted to the same uniform external pressure (provided cavity is free of forces).

Reviewer notes that, this way, it is proved, merely qualitatively, that previous exact particular solutions on pierced plates are consequence of the above quoted general property of elastic stress. See Kirsch [*ZVDI*, 1898], Kolossof [*Z. Math. Phys.* 1908], Wolf [*Z. Techn. Phys.* 1921], Supino [*Circ. Mat. di Palermo,* 1927; *Atti Accad. naz. Lincei R. C. Sci. Fis. Mat. Nat.*, 1927], and Sobrero [*Ric. Ingeg.* 1934]. The property explains also many results in the photoelastic field [see Cooker and Filon, "Photoelasticity," 1931, chap. VI]. Giulio Supino, Italy

2314. Pailloux, H., Statics and dynamics of rigid membranes (in French), *C. R. Acad. Sci. Paris* **234,** 14, 1430–1432, Mar. 1952.

An expression is given for the potential energy of a thin rigid membrane in terms of general curvilinear coordinates x^1, x^2, x^3 with metric $ds^2 = g_{ik}dx^i dx^k$, which should be of use in the discussion of problems relating to thin shells and membranes.

I. N. Sneddon, England

2315. Tumura, T., and Oosawa, M., Stress distribution in a rotating disk with conical shape (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17,** 61, 56–60, Nov. 1951.

Stress distribution in a rotating disk of hollow-cone shape of uniform thickness is solved by the usual shell theory. The result is as follows: (1) The value of stress at the center of the disk without central hole is not always greater than the value at the outer side. (2) In a special case, the value of stress at the center is equal to that at the outer side; it is lower, by 30%, than the value of stress in the center of a rotating plane disk of uniform thickness. (3) The stress at the center may possibly become the compressive stress in the case of some sharp conical angle. Teruyoshi Udoguchi, Japan

2316. Hirsch, R. A., The effect of a rigid circular inclusion on the bending of a thick elastic plate, *J. appl. Mech.* **19,** 1, 28–32, Mar. 1952.

See AMR **5,** Rev. 388.

2317. Szabó, I., Contributions to the theory of the axisymmetrically loaded thick circular plates, especially in case of elastic support (in German), *Ing.-Arch.* **19,** 6, 342–354, 1951.

Article is an extension of a previous publication. [See AMR 5, Rev. **1033.**] Author considers the problem of the loaded circular plate on an elastic foundation, assuming there is friction between plate and foundation and using the method developed in the former publication. Author indicates that the same method can be used to solve the problem of a thick circular plate supported at the edges and the problem of a thick circular plate clamped at the edges. J. W. Cohen, Holland

2318. Teichmann, F. K., and Wang, C.-T., Finite deflections of curved sandwich plates and sandwich cylinders, Fairchild Publ. Fund, Inst. aero. Sci. FF4, 14 pp., Jan. 1951.

The equilibrium differential equations are formulated for case where faces are treated as membranes, and stresses parallel to faces in core are neglected. Corresponding stress-displacement relationships are derived by use of method of complementary energy for nonlinear theory. Complementary energy $J = V + U - W$, where V is strain energy, W potential of external forces, and U work done by stress resultants in middle plane due to large deflection. Finally, modified Donnell's equation is given for case that effect of transverse normal stress deformability is negligible. For various special cases equations reduce to known relations. F. J. Plantema, Holland

2319. Malkin, I., Notes on a theoretical basis for design of tube sheets of triangular layout, *Trans. ASME* **74,** 3, 387–395, Apr. 1952.

To develop approximate analysis for problem of elastic bending of tube sheets of triangular pitch of holes, author first substitutes hexagonal grid of equal ligament area and then reduces solution to that of an isotropic plate of peculiar attributes, specifically an "effective" Young's modulus and Poisson's ratio which involve the ligament dimensions in addition to the properties usually understood by these terms.

Method is stated to have been in successful use by author for six years. As additional proof of its adequacy for engineering purposes, experimental results of deflection tests on sheets of rectangular outline are given and discussed in relation to theoretical developments. A. R. C. Markl, USA

2320. Nishihara, T., and Fujii, T., Stress in a semi-infinite plate having manifold notches (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17,** 61, 6–12, Nov. 1951.

Stresses in a semi-infinite plate having manifold notches in the edge which is subjected to a uniform tension are investigated, using the transformation function

$$z = w + \Sigma_1^n C_m e^{-mw}$$

where $z = x + iy$, $w = u + iv$, and C_m are chosen according to the boundary form. The stress function F is composed of fundamental stress function F_0 and additional stress function F_a, and each function is given by $\phi_0 + x\phi_1$, where ϕ_0 and ϕ_1 are harmonic functions. The results of numerical calculation show that the concentration of stress is decreased by the interference of notches as they are located more closely to each other.

Teruyoshi Udoguchi, Japan

Buckling Problems

(See also Rev. 2287)

2321. Libove, C., Creep buckling of columns, Fairchild Publ. Fund, Inst. aero. Sci., Prep. 353, 30 pp., 1952.

Theoretical study is made on creep buckling of pin-ended, H-section columns carrying constant load. The strain of the material is taken as composed of elastic strain and creep strain which varies exponentially with stress and K power of time. This nonlinear form of creep is taken because column in aircraft under consideration may have rather short service life. For simplicity, the lateral deflection shape is assumed to remain at all times sinusoidal. The equilibrium condition is satisfied only at the mid-height section; the strain rate is taken as a function only of the stress and strain and not of their histories. Results are given in charts showing the maximum length of time, the growth of stresses, strains, and deflections under given load prior to collapse. T. H. Lin, USA

2322. **Stein, M., and Mayers, J., Compressive buckling of simply supported curved plates and cylinders of sandwich construction,** *NACA TN* 2601, 34 pp., Jan. 1952.

This theoretical investigation is based on authors' small-deflection theory for curved sandwich plates [AMR **4,** Rev. 625] in which effects of deflections are included due to transverse shear. Two types of sandwich plates are considered—the corrugated-core type and the isotropic-core type. Results are given in the form of equations and curves. The effect of transverse shear stiffness, important in the range of practical dimensions, is discussed. Some results are compared with previous work. They are the same or more conservative.

W. L. Esmeijer, Holland

2323. **Lepik, Yu. P., Additional remarks on the cylindrical form of the loss of stability of plates, beyond the yield point** (in Russian), *Prikl. Mat. Mekh.* **15,** 1, 107–110, Jan.–Feb. 1951.

Author shows that the presupposition of the smallness of plastic deformation before the loss of stability, as compared to elastic deformation, is superfluous. George Herrmann, USA

2324. **Yen, K. T., Salerno, V. L., and Hoff, N. J., Buckling of rectangular sandwich plates subjected to edgewise compression with loaded edges simply supported and unloaded edges clamped,** *NACA TN* 2556, 66 pp., Jan. 1952.

Buckling stress for subject problem is determined from a system of equations which considers the bending rigidity of the faces as given by Hoff (see AMR **4,** Rev. **2437**). Values of critical stress are estimated as arithmetical means of solutions obtained by Leggett's method to establish lower bounds and by Galerkin's method to establish upper bounds.

Since the bending rigidity of the faces is usually neglected in sandwich buckling problems, it is of interest to compare results of subject reported with that of Seide [AMR **3,** Rev. 1252] who obtained an exact solution of this problem by treating the faces as membranes, to determine if refinement of the former leads to significantly different results. A direct comparison of the critical stress data of these two reports indicates slight differences for values of $h/t \geqslant 5$ (h core thickness, t face thickness) which appear to be between bounds of subject theory and also to within the scatter of the Forest Products Laboratory test data used by both reports to confirm theoretical results.

The significant result of the present theory, which is not presented by the authors, appears to be that, in cases where the core is weak in shear, the wave length associated with the critical stress is finite. This is an improvement over membrane theory where the wave lengths vanish for cores weak in shear.

George Gerard, USA

2325. **Müllersdorf, U., On the theory of plastic buckling** (in German), *Bauingenieur* **27,** 2, 57–61, Feb. 1952.

In 1947 Shanley pointed out [AMR **1,** Rev. 72] that an initially straight column subjected to continually increasing axial load is no longer stable against lateral deflection at loads exceeding the tangent modulus buckling load. Present paper investigates theoretically the deflection and maximum load of ideal two-flange columns on this basis. Although author was apparently unaware of it, this problem was previously treated by Lin, Duberg and Wilder, Pearson, and Cicala [AMR **4,** Revs. 1091, 1521 and 1987, 1097, and 1522, resp.]. Ingenuity appears in the analysis of the present paper but the results contain little that was not established by one or more of the papers cited.

Author closes with the observation that while of great theoretical importance, the changed outlook due to Shanley is of little practical significance because other uncertainties in engineering applications make it foolish to seek to come closer to the maximum load than is done by the more convenient tangent modulus formula of Engesser. Reviewer believes last point should be qualified; Duberg and Wilder have shown that, for materials such as stainless steel, the discrepancy between tangent modulus buckling load and maximum load may be quite large.

S. B. Batdorf, USA

Joints and Joining Methods

2326. **Soete, W., and Van Crombrugge, R., A study of the fatigue strength of welded joints,** *Weld. Res. Suppl.* **17,** 2, 100s–103s, Feb. 1952. (Extended abstract by Dr. G. E. Claussen, of "Étude de la Résistance a la fatigue des assemblages Soudés" published in *Rev. Soudure*).

The extended abstract summarizes the results of tests performed at the University of Ghent for the Belgian Standards Institute to procure quantitative information on fatigue tests of butt and fillet welds.

The base metal for all tests was steel having a tensile strength of 60,000 to 71,000 psi; carbon, 0.20% max; silicon, 0.20% max. Tests were made on two Amsler pulsators operating at 250–500 cycles/min and 30–100 ton capacity, respectively. Tests were discontinued after 2 million cycles from zero to maximum tension.

Fatigue strength of butt-weld specimens without restraint, machined and then stress-relieved, was 28,400–29,800 psi, or about 25% higher than either as-welded or stress-relieved specimens. Tests on butt welds under restraint seem to show that residual stresses have a negligible effect on fatigue strength.

Tests on fillet-welded cruciform specimens were performed to determine fatigue strength of fillet welds as a function of the dimensions of the weld. Such tests showed that fatigue strength remained at the same level regardless of the size of the fillet. The program of tests included fatigue tests on end fillet and side fillet specimens.

Reviewer believes that the results obtained are a valuable contribution to the knowledge of the fatigue strength in welded joints. Horacio O. Albano, Argentina

2327. **Structural adhesives, the theory and practice of gluing with synthetic resins,** London, Lange, Maxwell & Springer, Ltd., 1951, vi + 203 pp.

Aeor Research of Duxbridge, England, has for some time conducted a summer school on the technology of synthetic resin adhesives. Because attendants at the school come from various countries with resulting language problem, the lectures have been brought together and published in this volume, as explained by N. A. de Bruyne, one of the principal participants in the school. Subject matter, in general well and simply presented, covers the essentials of bonding various materials by means of synthetic resin adhesives. Wood and cellulose products, rubber and plastics, strength and testing, gluing techniques, high-frequency bonding, and the newer developments in bonding are adequately covered for the purposes of the summer course. The book is not intended to be as complete or detailed as de Bruyne and Houwink's "Adhesion and adhesives." A. G. H. Dietz, USA

2328. **Waters, E. O., and Williams, F. S. G., Stress conditions in flanged joints for low-pressure service,** *Trans. ASME* **74,** 1, 135–156, Jan. 1952.

The test program reported in this paper was undertaken to establish the critical levels of strain, stress, and deflection that exist in flanged joints having dimensions typical of those used

with steel pipe lines in potable water-supply systems. The investigation was limited to flanges of the low-hub and plain-ring types attached to the pipe by fillet welds.

From authors' summary

Structures

(See also Revs. 2245, 2293, 2302, 2305, 2318, 2322, 2361, 2367, 2372, 2373, 2515)

2329. **Roisin, V., Sariban, A., Zaczek, S., Method of fixed points applied to the calculation of hyperstatic systems** (in French), Centre Belgo-Luxemb. Inform. Acier, Bruxelles, 39 pp., 1951.

Each end constraint of a beam which is part of a frame is related to the bending stiffness of that beam by a "foyer." This is the point at which the bending moment is zero when a bending moment is applied at one end of the beam. Thus, two foyers correspond to the end constraint. On the basis of this definition, a simple scheme to the solution of frames and continuous beams is indicated. Diagrams are offered to help in the computations.

No new concept is added to the method of moment distribution as derived by H. Cross and the deductions already available. The paper is simple and well within the range of the engineer's use.

C. Riparbelli, USA

2330. **Bateman, E. H., Elastic stress analysis of multi-bay single-storey frameworks,** *Engineering* **172,** 4482, 4483; 772–774, 804–806; Dec. 1951.

Paper establishes the general solution of an arbitrary plane rigid frame composed of R straight prismatic members distributed over N bays. Employing matrix notation throughout, the author expresses the $2R$ bending moments at the ends of the members in terms of the applied forces, and the $3N$ redundant static quantities by means of the equations of statics. These equations, in combination with the $3N$ equations of minimum strain energy, lead to a system of $3N$ linear equations in the $3N$ redundants. The $3N \times 3N$ operational matrix $[Q]$ can be interpreted as an $N \times N$ matrix, in which each term q_{mn}, a 3×3 submatrix, is a function of the geometric and elastic properties of the mth and nth bays only.

The inversion of this operational matrix is facilitated by diagonalizing the principal submatrixes through a process which is analogous to the elastic center method in conventional structural analysis. The paper outlines a method of inversion through a series of successive approximations; in addition, it gives an exact solution for the example of a one-bay and a two-bay frame.

E. F. Masur, USA

2331. **Seiss, C. P., Viest, I. M., Newmark, N. M., Studies of slab and beam highway bridges: Part III. Small-scale tests of shear connectors and composite T-beams,** *Univ. Ill. Engng. Exp. Sta. Bull.* 396, 133 pp., Feb. 1952.

This bulletin reports experimental model tests of channel and T-shaped shear connectors used to develop interaction between a steel I-beam and a concrete deck of a highway bridge. The tests were (1) simple push-out shear tests, (2) static loading of a model beam and slab, (3) fatigue loading of a model beam and slab. Nearly all loadings were carried to failure of the connector. The data given are of value to designers of composite steel-and-concrete structures and to others interested in fracture of steel shapes.

L. E. Grinter, USA

2332. **Bleich, F., and Teller, L. W., Structural damping in suspension bridges,** *Proc. Amer. Soc. civ. Engrs.* **77,** Separ. no. 61, 31 pp., Mar. 1951.

Paper reports part of the work undertaken by the Advisory Board on the investigation of suspension bridges. It comprises: (1) Theoretical study of damping capacity of suspension bridges resulting from internal friction and from various sources of dry or Coulomb friction in the structure; (2) an account of an extensive laboratory study of frictional damping in structural members; and (3) a correlation of the theory with experimental data. Paper suggests that a substantial amount of additional damping can be provided without special damping devices if consideration is given in design to the potential sources of friction available in the floor system.

From authors' summary by R. E. Heninger, USA

2333. **Werren, F., Fatigue of sandwich constructions for aircraft,** *For. Prod. Lab. Rep., U. S. Dept. Agric.* no. 1559-J, 4 pp., 1 table, 4 figs., Apr. 1952.

A limited number of tests were made to determine the shear fatigue properties of a sandwich panel with facings of glass-fabric laminate and a core of alkyd isocyanate foamed in place. Repeated tests were made at a ratio of minimum to maximum loading of 0.1. The results of the tests indicate that the fatigue strength at 30 million cycles is about 30% of the static strength for the condition of loading used.

From author's summary

2334. **Norris, C. B., Strength of sandwich construction,** *Symp. Struct. Sandwich Constr., ASTM spec. tech. Publ.* 118, 46–53, 1952. $2.

A review of the work done at the Forest Products Laboratory on sandwich beams and columns is given in this paper. A number of modes of failure peculiar to sandwich construction are described, and their relations to allowable stresses and allowable loads are discussed.

C. T. Wang, USA

2335. **Upson, R. H., Phelps, G. M., and Liu, T-S, Equal-strength design of tension-field webs and uprights,** *NACA TN* 2548, 46 pp., Jan. 1952.

Report supplements previous NACA work on semi-empirical analysis of beams whose shear is carried in partial diagonal tension by a buckled web [AMR **1**, Rev. 634; **2**, Rev. 1501].

Practical design work is a long repetitive process even for experienced men. Present report develops simplified formulas and design procedures for rapidly determining satisfactory web and upright sizes. Even though restricted to beams with stiff caps and web-height thickness ratios between 400 and 800, method should prove very useful in aircraft design where many beams meet these restrictions.

Six test beams showed average ultimate shear loads differing from calculated ones by 12%. Designs were conservative in all but one case.

C. M. Tyler, Jr., USA

2336. **Yntema, R. T., and Milwitzky, B., An impulse-momentum method of calculating landing-gear contact conditions in eccentric landings,** *NACA TN* 2596, 60 pp., Jan. 1952.

An impulse-momentum method is presented for determining the changes in airplane angular and vertical velocities and landing gear vertical, drag, and side impulses during eccentric landings. Very little has been published on this important design problem. The methods developed allow a qualitative study for the critical design conditions arising from eccentric landings where one wheel, two wheels, or all gears hit the ground during first impact. The results obtained by using this method indicate

that eccentric landings may give rise to more severe landing impacts than symmetrical landings. A comparison of severity of impacts on auxiliary gears for both tricycle and quadricycle configuration is presented. J. E. Stevens, USA

2337. Sarkaria, G. S., Selection of an ideal section for gravity dams, *Civ. Engng. Lond.* **46,** 933–935, Dec. 1951.

Paper is continuation of author's earlier article [AMR **4,** Rev. 4434]. Theoretical procedure for selection of dam section is presented in which the triangular section is modified by providing in lower upstream part of dam a flatter slope, thus increasing stability against overturning and sliding by mobilizing additional weight of water on upstream face. Graphs are shown representing relationships between factors of safety against sliding and shear and the variables of section shape, i.e., upstream and downstream slopes and height of change of upstream slope. Volumetric quantities are also considered. A numerical example illustrates the procedure.

Wm. R. Schriever, Canada

2338. Johnson, A. I., Deformations of reinforced concrete, *Inter. Assn. Bridge struct. Engng. Repr.* from *Publications* 11, 253–290, 1951.

Author describes analytical study of deformations in reinforced-concrete members including prisms in tension, beams in flexure, and slabs. Expressions are given for computing elongation of reinforcement, compression of concrete, position of neutral axis, flexural rigidity, spacing and width of cracks. Effects of creep and shrinkage under long-time loading are studied analytically and experimentally in relation to redistribution of moment in restrained beams. Three restrained beams are tested to determine redistribution of moments both under sustained loading and at failure. C. P. Siess, USA

2339. Coepijn, W. C., Security against failure in cross sections of reinforced and prestressed concrete (in Dutch), *Ingenieur* **63,** 49, Bt. 81–Bt. 91, Dec. 1951.

For rectangular T- and I-shaped reinforced-concrete cross sections designed after the conventional modular ratio method, the bearing capacity under bending is computed after the theory of plasticity, taking into account the true stress-strain relations of both materials. The well-known fact is illustrated that the degree of safety against failure depends on the percentage of reinforcement and the material. Similar investigations are made for prestressed sections, and it is found that a higher concrete strength has a greater influence in prestressed than in reinforced concrete.

H. Craemer, Germany-Egypt

2340. Hawkes, J. M., and Evans, R. H., Bond stresses in reinforced concrete columns and beams, *Struct. Engr.* **29,** 12, 323–327, Dec. 1951.

Authors describe results of tests on reinforced-concrete columns and beams to determine the magnitude and distribution of bond stresses for various types of reinforcing steels. Pull-out tests show that distribution of bond stress along column is exponential function with a maximum at the loaded end. After initial slip, position of maximum moves along steel and its value remains sensibly constant.

Transverse loading tests on beams show that cracking of concrete causes bond stress over crack to rise rapidly with a maximum depending on bond qualities of reinforcement. Greater bond stresses were accompanied by larger deflections and wider cracks, which may limit allowable bond stress in practice. With mild steel, bond failure occurred only when yield point of steel had been reached. The latter result is due to the very high strength of concrete used and would not apply to the ranges of strengths commonly used in practice. Reviewer also considers that reference to extensive earlier investigations in various countries might usefully have been made.

G. G. Meyerhof, England

2341. Guerrin, A., Failure calculation of reinforced concrete (in French), *Ann. Inst. tech. Bât. Trav. publics (N.S.)* no. 234, 22 pp., Jan. 1952.

Paper deals with the ultimate-strength theory of designing concrete, analyzing the contribution of the concrete in tension to the ultimate resisting moment. Author begins with a review of the various stress diagrams which have been adopted by different authors, distinguishing between those which take into account the tensile strength of concrete and those which neglect it. Then he develops his thesis, emphasizing the tensile strength of concrete at failure as a real physical fact. The principal proofs to support his theory are: (1) The existence of a neutral axis implies the existence of uncracked concrete in tension. (2) According to tests of Saliger and Kuodis, cracks that affect the whole section are rare. (3) Neglecting the contribution of the concrete in tension, in the analysis of the tests of Gebauer, Ros, Kral, it is necessary to suppose the steel working at stresses above its ultimate strength.

Author concludes that the contribution of concrete in tension is of paramount importance for deep sections and slabs. Several numerical examples and curves support these statements.

In the discussion after the lecture, several speakers expressed their doubts about the author's assumptions, especially regarding the lack of experimental evidence for his propositions. It is necessary to underline the serious reply of M. Chambaud which mentioned nearly 500 tests made in the United States, which agree with the common assumption of neglecting the concrete in tension.

Reviewer makes the following comments concerning the basis and reasoning underlying the ultimate-strength theories. While some disagreement exists among the formulas of the different authors, there are two basic and common assumptions: The plane sections remain plane after the deformation; the ultimate strength of the two materials, concrete and steel, are expressed in terms of the axial strength of specimens of conventional shape and dimensions. But these simplifications are introduced to make possible the mathematical treatment of the problem, and are not an exact representation of the physical facts. The behavior of concrete in cylinder or cubes is not necessarily the same as in beams or in slabs; some departures may exist, but for practical purposes this is not of great importance. The same occurs with assumed stress pattern; obviously there are different shapes that can satisfy the requirements of the problem. It must be remembered that the choice of a stress pattern depends on practical considerations, and that all the formulas proposed contain arbitrary factors which are determined so as to make the theoretical values agree with those given by the tests. These coefficients can never be justified by experimental facts; hence it is meaningless to attempt to demonstrate that a formula is closer to the reality, justifying this with experimental data. The question may be asked whether there is any proof to show an excellent prediction of the ultimate structural capacity of a reinforced-concrete member, introducing varying coefficients determined solely to make the formulas conform to beam tests. Simplicity is perhaps the greatest requisite because the primary purpose of this theory is to provide a working tool for engineers. In this sense, Whitney has succeeded in developing simple design formulas, using the most restricted number of variables necessary to conform the results with the tests in a wide range. However, the greater the number of arbitrary constants, the more it is possible to obtain relationships that agree

better with the tests by a suitable choice of the constants. This is the case of Guerrin's relationships, but they are not practical because they are too complicated.

Regarding the contribution of the concrete in tension to the ultimate moment in beams, author's statements are far from being proved by experimental facts. On the contrary, evidence is presented that calculated values of tensile stress in reinforcing bars, with the ordinary theories, are sometimes lower than measured values for loads near the ultimate [AMR **5**, Rev. 1375]. It is interesting that the Russian code, which until 1946 utilized Steuermann's designing method which takes into consideration the tensile strength of concrete, changed this method into one which neglects it. In the field of slabs in accordance with Georges Steimann [*Inst. tecn. Const. Cemento*, no. 106, Jan. 1951], the results of Guerrin's relationships are in close agreement with the values obtained from the theory of breaking lines of K. W. Johansen, referred to data obtained by Gheler and Amos. In this case, formulas of Jager and Gebauer give an error of -56%.

This does not imply the exactness of both methods. Judgment must be reserved until satisfactory results have been obtained in a wide range of cases.

Cesar Augusto Sciammarella, Argentina

2342. **Acum, W. E. A., and Fox, L., Computation of load stresses in a three-layer elastic system,** *Géotechnique Lond.* **2**, 4, 293–300, Dec. 1951.

This is a further study of the stresses in multiple-layer road slabs. The first paper (1948) [AMR **3**, Rev. 887] treated the two-layer slab by two methods, one due to Burmister and the other due to Southwell. The present paper treats the three-layer system only by Burmister's method, since the relaxation technique proved too laborious. The loading is uniform over a circular area. The results are given in tables which are worked out for several ratios of slab thickness, diameters of loading, and moduli of elasticity. These tables give only the stresses along a vertical axis through the load point. The results appear to apply to a two-layer road slab since the third layer must be given properties to represent the soil or foundation.

L. E. Grinter, USA

2343. **Torroja, E., and Páez, A., New method of calculation of prestressed concrete [Nuevo método de cálculo del hormigón pretensado]**, Madrid, Inst. Técn. Constr. Cem., Apr. 1951, 294 pp. 50 ptas.

Theoretical portion presents a new method of designing for ultimate load. Basis for design is the simultaneous reaching of the maximum compressive stress in the concrete and the maximum tensile stresses in the steel and concrete. Curves approximating the true stress-strain curves are used in the derivations of expressions for design purposes. Loss of tension in reinforcement, due to various causes, is discussed. The various design expressions, which are summarized in orderly fashion, include a factor of safety for which an arbitrary value may be substituted. The second portion describes (1) various means of tightening and anchoring the reinforcement, and (2) some completed structures. The reviewer believes the method will bear some consideration because of the trend toward ultimate design in reinforced-concrete structures and the requirement of checking prestressed-concrete structures by the ultimate load theory.

Robert B. B. Moorman, USA

2344. **Abeles, P. W., The design of prestressed concrete beams for working conditions,** *Concr. Constr. Engng.* **47**, 3, 75–86, Mar. 1952.

Assuming rectilinear stress distribution under ordinary loading conditions, author covers the design of the following types of beams: (1) T-beams with posttensioned wires or bars; (2) I-beams unsymmetrical about the horizontal axis; (3) symmetrical I-beams with pretensioned wires.

Ai-ting Yu, USA

2345. **Ewell, W. W., Okubo, S., and Abrams, J. I., Deflections in gridworks and slabs,** *Proc. Amer. Soc. civ. Engrs.* **77**, Separ. no. 89, 22 pp., Sept. 1951.

A method of determining deflection patterns of plates or slabs subjected to normal loads by transformation into an equivalent grid system. The procedure consists of the following steps: (1) Division of the plate or slab into rectangular strips. (2) Determination of relative stiffness and moment distribution factors. (3) The vertical displacement at joints where deflection produces a clearly defined moment and torque pattern and the resulting fixed end moments. (4) Conversion of fixed-end moments into bending and twisting moments by a moment and torque distribution process. (5) Computation of reactions at each joint in terms of the unknown displacements. (6) Computation of moments and torques at each point which were expressed by the distribution process. Essentially a numerical process resulting in the solution of only one series of simultaneous equations. Several examples are included.

Clarence B. Matthews, USA

2346. **Barta, J., Calculation of the vibration frequency of towers,** *Acta Techn. Hung., Budapest* **2**, 2/4, 491–497, 1952.

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 2379, 2382)

2347. **Thayer, G. B., Plastics literature references,** *Mech. Engng.* **74**, 4, 309–311, Apr. 1952.

Literature selected for the mechanical engineer, July 1950–June 1951.

2348. **Birger, I. A., Certain general methods of solution of problems in the theory of plasticity** (in Russian), *Prikl. Mat. Mekh.* **15**, 6, 765–770, Nov./Dec. 1951.

Author outlines three methods of solving, by successive approximations, problems in formal plasticity theory (of the finite or "deformation" type). In each method, the actual body A is replaced by a related elastic body B. In method 1, the strains in B are kept the same as in A by application of volume and surface forces not present in A. In method 2, the stresses in B are kept the same as in A by superposition of additional strains upon those corresponding to Hooke's law. In both these methods, B is elastically linear. In method 3, the elastic parameters of B vary with strain so as to keep both the strains and the stresses the same as in A. The initial approximation is obtained by neglecting the additional forces in method 1, the additional strains in method 2, and the nonlinearity in method 3. No examples are given or cited of specific problems solved by these methods. Proof of convergence is not attempted, but is stated to have been given for method 1 by Panferov [AMR **3**, Rev. 691].

William Fuller Brown, Jr., USA

2349. **Prager, W., On the boundary-value problems of the mathematical theory of plasticity,** *Proc. intern. Congr. Mathematicians* 1950, **2**, 297–303, 1952.

The progress of plastic deformation for an ideally plastic material is described from its first beginning through the stage of contained plastic deformation to impending plastic flow. The plane strain problem of the tensile specimen with semicircular

grooves, considered numerically by Allen and Southwell [AMR 4, Rev. 203], is discussed from the limit-design point of view. A simple lower bound is found which proves that the force which first produces a bridge of plastic deformation all the way across the specimen is below the load required for impending flow (collapse). D. C. Drucker, USA

2350. Kuntze, W., Yield condition under nonuniform stress distribution (in German), *Bauingenieur* **26**, 12, 357–362, Dec. 1951.

Paper is concerned mainly with five theories for yield of a beam, with linear stress distribution varying from zero to $\sigma_{\max}$, compared with the yield in uniform tension σ_s. They assume (a) no effect of stress gradient, $\sigma_{\max} = \sigma_s$; (b) mean stress equals σ_s; (c) average elastic strain energy the same as uniform tension; (d) equality of plastic and elastic moments; (e) equality of moments based on the radius arm that divides the force equally, rather than that based on the true first moment for the force distribution. The latter concept is discussed at length, and is stated to give best agreement with experiment. The influence of upper and lower yield points is discussed.

Reviewer believes that the extended discussion of moment equilibrium conditions and virtual work to be valueless, for laws of this type cannot be more than empirical. The theory of the influence of stress gradient on yield condition demands a much deeper theoretical attack. The elastic energy theory seems to be incorrect, $\int \epsilon d\sigma$ being used for $\int \sigma d\epsilon$ with a nonlinear law; moreover, the linear stress distribution assumed would not be produced by bending in this case. E. H. Lee, USA

2351. Roberts, C. S., Carruthers, R. C., and Averbach, B. L., The initiation of plastic strain in plain carbon steels, *Trans. Amer. Soc. Metals* **44**, 1150–1157, 1952.

The initiation of plastic deformation has been studied in a series of plain carbon steels with various metallographic structures. It has been found that there is a separate elastic limit at an even lower stress than the lower yield point, and there is a region of continuous plastic flow prior to the discontinuous yielding. The effect of prior plastic deformation has been determined, and the elastic limit of steels has been correlated with the mean ferrite path available for plastic deformation.

From authors' summary by W. M. Shepherd, England

2352. Gerard, G., and Wildhorn, S., A study of Poisson's ratio in the yield region, *NACA TN* 2561, 30 pp., Jan. 1952.

Aluminum alloys 24S-T4 (rolled) and 14S-T6 and 75S-T6 (extruded) were tested in tension and compression along each of the three principal axes. The data "indicate that tensile loading in the yield region is accompanied by a permanent decrease in volume, whereas compression results in a permanent increase in volume. These data were subsequently checked by density measurements on several specimens."

F. R. N. Nabarro, England

2353. Hedström, B. O. A., Flow of plastic materials in pipes, *Indust. Engng. Chem.* **44**, 3, 651–656, Mar. 1952.

Paper deals with engineering calculations for flow of ideal plastics (Bingham fluids) through pipes in laminar and turbulent regions. Nomograms are presented for McMillen's equations [*Chem. Engng. Progress* **44**, p. 537, 1948]. Author concludes Fanning friction factor and Reynolds number are applicable to Bingham fluids, with "plastic viscosity" replacing Newtonian viscosity. Method is illustrated with data of Wilhelm, et al. [title source, **31**, p. 622, 1939] on cement rock suspensions. Reviewer feels method for finding flow behavior in laminar region is inferior to differentiation method of Saal and Koens [*J. Inst. Petrol. Tech.* **19**, p. 176, 1933].

Irvin M. Krieger, USA

2354. Andrade, E. N. da C., Viscosity and plasticity, New York, Chemical Publishing Co., 1951, 80 pp. $2.25.

Book is based on lectures originally presented by the author for industrial rheologists. It sketches general principles from which an understanding of the behavior of flowing liquids and solids is obtained. The scope of the book is indicated by its chapter headings: I. Nature and theories of liquid viscosity; II. The flow of simple liquids, suspensions, and gels; III. The flow of solids.

In chapter I, besides explaining classical concepts of rheology, such as Newtonian liquids, Maxwell's relaxation, analogy between elastic and viscous behavior, author puts forward concepts of hydrodynamics, such as Reynolds number, laminar and turbulent flow, vorticity, etc. A critical presentation of Eyring's reaction rate theory and comparison with author's model of associating molecules which transfer momentum during the process conclude the chapter.

Chapter II describes methods of measuring viscosity (including Derjaguin's blow-off method) and, in connection with this, gives examples of non-Newtonian behavior. Instead of precise definitions and systematic discussion, a clear explanation is given of the behavior of lyophobic and lyophilic sols, thixotropy, dilatancy, fals body, etc. Eisenschitz's investigation on lyophilic and Einstein's on lyophobic suspensions are described in detail.

Chapter III consists of discussion of elastic, plastic, and viscous behavior, Bingham solid, and metal flow.

Reviewer feels that author's purpose was best achieved in chapter II. Presentation is simple and clear, book is most readable. The many numerical values given, the 32 explanatory drawings, author's lucid style, and the substitution of mathematical treatment by experimental approach make the book a unique contribution. V. G. Szebehely, USA

2355. Thrower, E. N., A servo-controlled constant-stress rheometer, *J. sci. Instrum.* **29**, 3, 91–93, Mar. 1952.

Constructional details are given for a constant-stress rheometer suitable for the investigation of the rheological properties of highly viscous materials. Cylindrical specimens, up to 2 in. long, are compressed under either a constant load or a constant stress. For this compression a servo system is employed which eliminates frictional errors and makes possible the automatic recording of both deformation-time and recovery-time curves, without interfering with either process. The instrument is suitable for investigating a range of from 10^7 poises to 10^{12} poises. From author's summary

2356. Cicala, P., On the Batdorf and Budiansky theory of plastic deformation (in Spanish), *Rev. Fac. Cienc. exact. fis. nat., Univ. Nac. Córdoba* **13**, 2, 401–415, Apr./June 1950.

Paper studies analytically certain implications of the slip theory of plasticity [AMR **2**, Rev. 1264]. The slip theory of plasticity is based on the assumption that plastic strain is the tensorial resultant of infinitesimal slips in every direction on every plane section through the material element; the magnitude of a given component of slip is given by a certain characteristic function of the largest value of the corresponding projected shear stress that has occurred during the history of loading. The plastic shear strain tensor is then expressed in terms of a triple integral involving the characteristic slip function and trigonometric functions related to the orientations of the infinitely many planes and directions of slip.

Author considers the strains resulting from a small variation in the stress tensor starting from a state of uniaxial stress in the plastic range. If the increment of the axial stress is sufficiently large relative to the increments of the other components of the stress tensor, then slip will continue to grow in all of the directions on all planes where it has already occurred due to the original uniaxial stress. For this situation, by means of ingenious reductions of the pertinent triple integrals to forms involving elliptic functions, author shows that the resultant plastic strain increments may be expressed directly in terms of the properties of the uniaxial stress-strain curve, without explicit determination of the characteristic slip function. In particular, for the case of variations only in axial stress $\delta\sigma_z$ and shear stress $\delta\tau_{xz}$, it is proved that when $\delta\sigma_z/\delta\tau_{xz}$ is large enough, the apparent initial shear modulus predicted by the slip theory is identical to that predicted by the Hencky (deformation) theory. For lower values of $\delta\sigma_z/\delta\tau_{xz}$, the initial shear modulus increases, becoming the elastic value predicted by the Prandtl-Reuss (flow) theory when $\delta\sigma_z/\delta\tau_{xz}$ is a sufficiently large negative value, for which there is no increment of slip on any plane. At intermediate values of $\delta\sigma_z/\delta\tau_{xz}$, the characteristic slip function is needed for the calculation of the plastic strains; author deduces an integral equation from which this function can be found from the uniaxial stress-strain curve.

The results concerning initial shear modulus (which had been indicated qualitatively by Batdorf [AMR **3**, Rev. 1924]) are significant in that they afford a possible explanation for the success of deformation theories in the analysis of plastic buckling of plates. Author suggests simple experiments on thin-walled cylinders to attempt to verify the predictions of the slip theory that he calculates.

Reviewer's note: Experiments similar to those recommended by the author have been carried out at the Langley laboratory of the NACA with results that do not appear to bear out these predictions; see the forthcoming *Proc. First U. S. nat. Congr. appl. Mech.* Bernard Budiansky, USA

2357. Hastewell, L. J., and Ritson, F. J. U., A servomechanical apparatus for the investigation of the properties of visco-elastic materials, *J. sci. Instrum.* **29**, 1, 20–23, Jan. 1952.

Authors describe a coaxial-cylinder rotational viscometer embodying an ingenious servomechanism which causes the sample to be subjected to an arbitrarily chosen constant stress, instead of the more common constant shear rate. Constructional dimensions of the instrument are not given. The servomechanism and recording system possess fairly rapid damped response, allowing for the investigation of viscous or pseudoplastic materials of an effective relaxation time upward of one second. Maximum shear rate reported is about 0.2 sec^{-1}. Method of analyzing flow results (for a gum tragacanth solution) corresponds to that employed by Robinson, Ruggy, and Slantz [*J. appl. Phys.* **15**, 343–351, 1944] for flow of polymethyl methacrylate. A kind of relaxation experiment, in which sample is suddenly sheared, then allowed to recover (e.g., see M. Mooney, *J. Coll. Sci.* **1**, 195–208, 1946), yields remarkably good agreement with a relaxation curve computed (evidently by method of Maxwell), using parameters obtained by constant stress experiments. J. T. Bergen, USA

2358. Schwarzl, F., Methods of approximation in the theory of viscoelastic behavior. I, II (in German), *Physica* **17**, 9, 10; 830–840, 923–929; Sept., Oct. 1951.

Author deals with a generalized Boltzmann-Becker theory of elastic aftereffect in which time-dependent linear strain-stress relations are postulated [cf. T. Alfrey, "Mechanical behavior of high polymers," Interscience Publishers, New York, 1948, appendix II]. The mathematics is drastically simplified by substituting rectangular graphs for various analytical functions and employing a simple formula in the place of the Laplace inversion. Resulting equations are well suited for graphical or numerical evaluation of experimental stress-time and similar functions. Author derives simple relations between flow curves, relaxation curves and the frequency dependence of the complex elastic coefficient. Since even the original theory is somewhat idealized and the author's simplifications are not justified on mathematical grounds, the value of the results depends entirely on the experimental test on which, however, nothing is said. R. Eisenschitz, England

2359. Vigness, I., Investigation of stress-strain relations of metal wires by electrical resistance changes, *J. appl. Phys.* **23**, 1, 43–47, Jan. 1952.

A method is described by which the stress-strain curve of a wire tensile specimen can be derived from measurements of its changes of electrical resistance as a function of strain. The method can be applied only to a selected group of metals that have been experimentally shown to be suitable. A material is suitable when its resistance changes per unit of elastic and per unit of plastic strain are each constant and different from each other. Besides, the method assumes uniform strain throughout the length of the specimen without necking.

Thin wires (about 0.001-in. diam) of a platinum-iridium alloy (95% Pt, 5% Ir), of manganese, iron, soft constantan, and nickel have been investigated. It has been shown that the plastic gage factor without releasing the load $G_P = \Delta R/R\epsilon_P$ and the plastic gage factor $G_{PR} = \Delta R/R\epsilon_P$, obtained by stretching the wire beyond its yield point and releasing the load, have the same value within experimental accuracy for the first three materials. If the elastic gage factor $G_E = \Delta R/R\epsilon_E$, the plastic gage factor G_P, and the elastic modulus E are known, the stress-strain curve is derivable graphically or from the analytical expression $\sigma = E/G_E - G_P \cdot (\Delta R/R - G_P \cdot \epsilon)$, where ϵ means the total strain.

The gage factors of constantan wire are nearly equal over both elastic and plastic range, a fortunate coincidence when a metal is used as a bonded-wire strain-gage element. Nickel possesses no linear relation between resistance and strain in the elastic region. This anomaly is related to its magnetostrictive properties. Therefore, constantan and nickel are unsuitable materials for the described method, which appears to be best suited to studying stress-strain relations over a wide range of strain-rate and temperature conditions. Franz Wever, Germany

Failure, Mechanics of Solid State

(See also Revs. 2369, 2371, 2383)

2260. Thielsch, H., Thermal fatigue and thermal shock, *Weld. Res. Counc. Bull. Series* no. 10, 24 pp., Apr. 1952. $1.

Review of published and unpublished information on characteristics of thermal fatigue and shock; effects of stresses, design, structure, carbon migration, intergranular oxidation, phase transformations, welding, etc.; occurrence of thermal fatigue and shock in various applications; testing procedures used to evaluate thermal fatigue and shock.

From author's summary

2361. Symonds, P. S., and Neal, B. G., The interpretation of failure loads in the plastic theory of continuous beams and frames, *J. aero. Sci.* **19**, 1, 15–22, Jan. 1952.

It is shown that the usual simple theory for computing plastic failure loads may, in certain cases, give misleading results; viz.,

if large deflections already occur before the last plastic hinge, formation of which is assumed to define plastic failure load, develops. Procedure is given for approximate calculation of deflections corresponding to plastic failure loads of simple theory. Results will be accurate when effects of the two simplifying assumptions made approximately cancel each other: (1) Additional flexibility (over elastic flexibility) due to spread of plastic zones is neglected; (2) strain-hardening at plastic hinges is neglected. Calculation may be made directly from collapse analysis without requiring successive elastic solutions as various plastic hinges develop, but it requires knowledge of which plastic hinge will form last. If the latter is not obvious, all likely cases must be analyzed and the correct one is that leading to largest deflection.

Examples are given for continuous beams and a two-story portal frame. F. J. Plantema, Holland

Design Factors, Meaning of Material Tests

(See also Rev. 2296)

2362. Defay, A., Safety in structures (in French), *Ossature métall.* **17**, 2, 85–88, 1952.

This expository paper advocates the use of a probabilistic method for defining a safety factor. Method is entirely experimental and is illustrated with reference to a bar in simple tension. A given extension is taken as desired, several experiments are made, and a frequency curve plotted of the necessary force. On the same axes, a frequency curve of the force necessary to produce failure is plotted. The area under both curves simultaneously is proportional to the probability of failure before the given strain is produced.

Frequency curves are assumed Gaussian, and usual interpretation of probability of deviation from the mean is given. Since homogeneous materials are more uniform in behavior, frequency curves will have sharp peaks at mean. Thus a very nearly homogeneous material may be "safer" than another material with higher mean failure load. Some observations are advanced on optimum design based on cost and probability of failure.

Author claims method is better than conventional one because it is more realistic, and hence leads to most economical construction. Nothing is said of cost of experiments, but, in reviewer's opinion, this might well outweigh advantages of the method. Certainly a ten-story building cannot be repeatedly tested to failure. Method would seem to have merit only if experimental data on simple tests can be extended by theory to complex structures.

Philip G. Hodge, Jr., USA

Material Test Techniques

(See also Revs. 2297, 2377, 2378, 2382, 2392)

2363. Locati, L., Programmed fatigue tests; variable amplitude rotating bending tests (in Italian), *Metallurgia ital.* **44**, 4, 135–144, Apr. 1952.

The course of stresses exerted on working elements differs greatly from the one expected from a sine wave function; there is either the superimposition of different frequencies or the variation of amplitude in the cycle.

Following the review of the various studies carried out to date on the "nonsinusoidal fatigue," a new instrument is described by which the rotating bending tests with stresses varying in time can be carried out according to any fixed program. Tests carried out on some technical materials are reported and the results therefrom are represented in the form of "training."

Since the training effect disappears for seasoning treatments at even very low temperature, it is supposed to be due to the presence of unstable internal stresses caused by creeps concerning only the internal part of the grain, easily annulled by recovery phenomena. From author's summary

2364. Shaw, M. C., Abradoflex—abrasion resistance tester, *ASTM Bull.* no. 180, 49–52, Feb. 1952.

Heavy textile materials, such as asbestos fabrics used for safety clothing and draperies, are subjected to severe flexural abrasion in service. Conventional laboratory abrasion tests do not give good correlation with service results. Author describes a tester combining flexing and abrasion. Loops of fabric are fastened at their ends to a cylinder. The loops are rotated against a smaller, abrasive-covered roll. Loop wraps around smaller roll and passes between it and the main cylinder, providing flexing and abrading action. Three loops are mounted on main cylinder and act against abrasive rolls. A fourth acts against a steel roll to evaluate the effect of flexing alone. Cylinder rotates at 100 cpm with small rolls at 50 cpm. One specimen is removed at $^1/_2$ hour, one at one hour, and one at three hours. All four are tested for tensile strength retention. An index number which shows percentage of strength retention of the abraded and flexed specimens for each of the three periods is obtained. A flexural rating for the fourth specimen is given in terms of the percent loss in strength after three hours.

Author tabulates results for a wide variety of asbestos fabrics, differing widely in weave, weight, and construction. Correlations between fabric geometry and Abradoflex rating appear to be possible, but author draws no conclusions of this nature.

Rogers B. Finch, USA

2365. Krautkrämer, J., Nondestructive material testing with ultrasonic impulses (in German), *Werkstoffe Korr.* **3**, 4, 125–128, Apr. 1952.

A general description of ultrasonic crack detection introduces the paper. The application of the ultrasonic reflection method to thickness control, the detection of longitudinal cracks in pipes by the oblique incidence method, and the use of the same for the test of weldments are described. Author developed double-action twin heads for oblique incidence tests (an improvement over the Sperry and Hughes methods) which distinctly indicate if the piece is crack-free, and locate easily the discontinuities, if there are any. The oblique incidence heads are fixed to Plexiglas blocks which can be adjusted to different curvatures of the test pieces. D. Vasarhelyi, USA

2366. Josefsson, Å., and Kula, E., Internal friction measurement on iron wires of commercial purity, *J. Metals* **4**, 2, 161–165, Feb. 1952.

Paper describes the use of low-frequency internal friction measurements to study the presence of impurities in commercial steels, specifically nitrogen, manganese, and phosphorus. Internal friction vs. aging-time curves are also presented for various heat treatments with photomicrographs.

Reviewer believes paper to be of practical value with regard to materials studied. Authors follow techniques and interpretations developed by Zener and Kê, and hence, probably used the torsion-pendulum method. Specimens were wires 0.08-mm diam. Herbert I. Fusfeld, USA

2367. Yang, C. H., Lynn, S. B., and Johnston, B. G., Residual stress and the yield strength of steel beams, *Weld. J.* 31, 4, 205s–229s, Apr. 1952.

Paper concerns effect of residual stress, and stress concentra-

tions, upon the yield strength of beams, frames, and columns of structural steel. The commonly used (but seldom verified) assumption that plane sections remain plane in plastic action of beams is shown to give considerable error in the prediction of load-deflection or moment-curvature relations. The assumption of uniform distribution of plastic strain along a length of beam under uniform moment is shown to be incorrect. Instead, it is shown that narrow plastic zones form at discrete spacings. The residual stress due to nonuniform cooling after rolling is shown to have a more pronounced effect in reducing a yield strength which is dependent upon allowable deflections than a yield strength dependent upon an allowable stress. The buckling load for a column is also shown to be significantly reduced by residual stresses. Paper gives extensive experimental and theoretical results of interest to persons concerned with the development of plastic design methods. Stanley U. Benscoter, USA

2368. **Dodd, R. A., Some observations on the determination of residual stresses in cylindrical metal bodies by Sachs' boring method,** *Metallurgia, Manchr.* **45,** 269, 109–114, Mar. 1952.

Experimental method is suggested to obtain reliable measurement of residual stresses by reducing errors normally introduced by electrical-resistance strain gages and by machining. Review of measurement techniques shows necessity of making measurements over shortest period of time. Special tools, to prevent vibration and permit free use of a coolant, reduce machining stresses in thick wall cylinders. X-ray methods also show good correlation for Sachs' method for cylinders with length/diameters as low as 1:1. It is believed methods and results further verify Sachs' boring method. C. R. Freberg, USA

2369. **Pearson, J., and Rinehart, J. S., Deformation and fracturing of thick-walled steel cylinders under explosive attack,** *J. appl. Phys.* **23,** 4, 434–441, Apr. 1952.

Tests have been conducted on annealed heavy-walled cylinders of low-carbon (1020) steel internally loaded by explosive charges. The purpose of these tests was to obtain basic information on the manner and type of fracturing and plastic flow obtained by extremely high pressures acting for short durations. It was observed that all of the cylinders tended to fracture in long fragments in the same basic pattern, but with variations because of wall thickness and manner of loading. Experimental results indicate that the radial cleavage type of fracture is initiated within the cylinder wall and propagated to the surface, while the shear type fracture associated with the inner portion of the cylinder wall appears as an independent energy-relieving process.

Microstructure analysis of the cylinder fragments shows a definite relationship between the type of fracture and the amount of distortion of the grain boundary. Considerable shock twinning is present in the cylinder fragments, and the grain structure near the inner surface shows severe distortion and flow. Strain measurements indicate that considerable plastic strain occurs during the explosive loading, and cleavage fractures were always observed to occur in association with considerable plastic deformation. From authors' summary

Mechanical Properties of Specific Materials

(See also Revs. 2294, 2301, 2352, 2363, 2364, 2367, 2390)

2370. **Marin, J., Engineering materials, their mechanical properties and applications,** New York, Prentice-Hall, Inc., 1952, ix + 491 pp. **$8.70.**

The stated aim of this book is to provide both students and designers with enough information to use materials intelligently in structures and machines that undergo widely varying operating conditions. Although the cover title is simply "Engineering materials," it should be noted that particular emphasis has been placed on methods of calculation for design. For example, the first part of the text, after presenting the standard definitions of mechanical properties, discusses various theories of flow and fracture under combined stresses, methods for determining fatigue strength under various systems of stress, elastic impact, and design for creep.

The second part of the text, which perhaps more nearly fits the cover title, presents a short general discussion on the structure of materials and the control of their properties. It then describes the manufacture, use, and properties of specific materials.

The third part of the text consists of a survey of materials-testing machines and strain gages. In addition to simple stress machines, combined stress, creep-testing, impact-testing, hardness-testing, and numerous fatigue-testing machines are described. In the portion covering strain gages, the well-known mechanical, optical, and electrical types are briefly described.

The book's particular merit is that it brings design problems that have become important in recent years to the attention of the student. In this sense, the book, in addition to covering engineering materials, surveys some of the design theories that have been advanced in recent years. In a book of so broad a coverage, mention of all existing theories on each subject is, of course, impractical; however, in the discussion of plastic flow where the deformation theory is presented, the incremental theory is certainly worthy of mention. In the section on creep behavior in which primary stage creep is neglected to accomplish bending and relaxation solutions, some reference should be made to the recent attempts to include the primary stage creep in these solutions.

On the whole, this is a well-organized clearly written book and it successfully accomplishes the aim of the author.

A. D. Schwope, USA

2371. **Roš, M., Fatigue of metals. Bodies with notches. Results of tests in the LFEM (1948–1950)** (in Italian), *Metallurgia ital.* **63,** 12, 512–520, Dec. 1951.

See AMR **5,** Rev. 720.

2372. **Krisch, A., Creep tests with steel wire at room temperature** (in German), *Arch. Eisenhüttenw.* **22,** 9/10, 313–316, Sept./Oct. 1951.

Six steel wires with different chemical compositions and heat treatments, with 222 to 328-ksi tensile strength, were subjected to creep at room temperature in a special weight and lever machine. The total strain of the wire was maintained approximately constant during a test by moving the weight manually after each 0.1-mm displacement of the lever. The initial stress in the wires varied from 45 to 75% of the corresponding tensile strength. The stress decrease in the wires was observed continuously during 1000 hours. The final stress decrease increased with increasing initial stress and varied from about 1.5 ksi (for the 45% test) to about 5 ksi (for the 75% test).

At low stresses, there was hardly a difference between the different wires. The higher tensile strength wires showed a distinct superiority for stresses of about 170 ksi only. The stress decrease with time obeyed for all tests a logarithmic law of the form $\sigma = A - B \log t$. This law was even found to represent satisfactorily the results of two additional tests wherein the wires were loaded to 97 or 95% of their tensile strength.

Extrapolation by means of this law would give for the stress decrease after 100 years values which are still permissible in

prestressed concrete; but this extrapolation involves many uncertainties. Ch. Massonnet, Belgium

2373. Smith, D. W., Whitman, J. G., and Cottrell, C. L. M., Stress-strain relationship of a high-tensile weldable structural steel, *Metallurgia, Manchr.* **45,** 270, 169–172, Apr. 1952.

The high-tensile manganese-molybdenum steel employed in bridge construction shows, in the hot-rolled or normalized condition, a low limit of proportionality which could affect adversely the strength of the bridges and the interchangeability of parts. It is not possible to raise the limit of proportionality for both tension and compression loading by prestressing in either sense.

Greatly improved limit of proportionality and notched-bar impact figures are obtained by tempering at 600 C. The low limit of proportionality in relation to yield stress is avoided if carbon and molybdenum content are reduced, but the strength of the steel is then inadequate. A 0.02% proof stress of 20.5 tons/sq in. is generally (but not with complete certainty) obtainable in the hot-rolled condition in steel containing carbon 0.18, manganese 1.35, and molybdenum 0.25%; but a higher value cannot be obtained consistently without tempering.

From authors' summary

2374. Kollmann, F. F. P., Selection, testing and properties of wood and laminated wood for highly-stressed structural parts, *J. aero. Soc. India* **4,** 1, 1–26, Feb. 1952.

Mainly from the viewpoint of light construction, wood and wood-based materials have proved advantageous. Careful selection based on growth properties and annual ring structure is a prerequisite. Knots generally must be excluded, also all discolorations by fungi, even by blue stain. The importance of mechanical tests is significant. Considerable data on the modulus of elasticity, the tensile, compression, buckling, and bending strengths are presented. In the case of impact work and the determination of torsion and shear strengths, many points still need to be investigated. With extensive data on frequency distribution of test results, an evaluation of properties for both solid wood and laminated wood is presented. The dependence of the mechanical properties of wood on density, moisture, and grain angle, and of laminated wood on the method of preparation is shown in a number of charts. J. Neils Thompson, USA

2375. Worley, W. J., and Findley, W. N., The effect of temperature on the creep and recovery of a melamine-glass fabric laminate, *Proc. ASTM* **50,** 1399–1412, 1950.

Results of creep tests of a melamine-glass fabric laminate at temperatures of 40, 77, and 130 F are reported. The relative humidity of the air surrounding the specimens was maintained at about 50% by improved apparatus described. Creep was measured from specimens tested at several different stresses for each of the three temperatures. Dimensional changes under no stress and changes in weight of the material were also recorded.

Correlation of the data with the activation-energy theory of creep is examined by means of a new method of analysis. It was observed that the creep resistance of this material was much greater than that of the canvas laminate and rayon laminate previously reported [see AMR **2,** Rev. 65], and its temperature sensitivity was much less than that of the canvas laminate.

The problem of obtaining accurate creep data on plastics is discussed. From authors' summary

2376. Cleereman, K. J., Karam, H. J., and Williams, J. L., Heat distortion of polystyrene, *ASTM Bull.* no. 180, 37–44, Feb. 1952.

The ASTM heat-distortion test consists of applying a load at the center of a horizontally supported plastic bar. The bar is then heated at a uniform rate until a specified distortion has occurred. The temperature at which this occurs is called the heat-distortion temperature. This paper reports on an investigation of the effect of annealing during conditioning on the heat-distortion temperature.

For the three types of polystyrene tested, it was found that annealing increased the heat-distortion temperature. In general, the heat-distortion temperature was a function of annealing temperature and time. However, a maximum limiting heat-distortion temperature was reached in all three cases. This limiting value is dependent only on the type of material used. Authors claim that this value is a reliable index for control purposes and, from a practical viewpoint, it represents the uppermost temperature at which fabricated articles made from a given plastic may be considered for use.

Authors also report, however, that the rate of distortion of the annealed specimen was more rapid than that of the unannealed bar. In view of this fact, reviewer thinks that it would be interesting to compare some constant-load creep-test results of annealed and unannealed specimens. It is quite possible that annealed fabricated articles under sustained loads would fail from distortion at temperatures far below the heat-distortion temperature determined by the procedure recommended here.

Yoh-Han Pao, USA

2377. Giauque, W. F., Geballe, T. H., Lyon, D. N., and Fritz, J. J., Some properties of plastics and the use of plastic apparatus at low temperatures, *Rev. sci. Instrum.* **23,** 4, 169–173, Apr. 1952.

Some properties of methyl methacrylate plastic have been investigated at low temperatures to supply design data needed for the construction of plastic apparatus. The diffusion rates for air and helium have been measured at temperatures from 260 to 300 K. The diffusion coefficient k [expressed as cm³ (N.T.P.) sec⁻¹ (cm Hg⁻¹)] is given roughly by the following relation: $\log_{10}k = [(-2.85 \times 10^3)/T] - 1.68$ for air, and $\log_{10}k = [(-1.28 \times 10^3)/T] - 4.95$ for helium.

The change in length of a 30-cm rod has been measured at 298.7, 273.16, 194.7, 77.3, and 20.4 K by comparison with quartz using a depth micrometer gage. The coefficient of expansion is tabulated as a function of temperature, increasing from 0.6×10^{-5} at 20 K to 8.0×10^{-5} at 300 K. Hysteresis and geometrical distortion accompanying cooling are shown to be very small. Complex apparatus containing many seals, including various sorts of electrical leads, has been shown to be vacuum-tight when tested in liquid helium II. A method of connecting a vacuum line at room temperature and a plastic apparatus at low temperatures by means of a metal tube liner-diluted rubber-cement seal is described. The adsorption of helium gas has been measured at 4.2 and 1.2 K and at pressures from 4×10^{-4} to 4×10^{-1} mm. From authors' summary

2378. Brown, G. L., Fabric testing machine, inclined plane type, *J. Text. Inst. Trans.* **43,** 3, T119–T122, Mar. 1952.

This note describes a machine designed and constructed at Defense Research Laboratories to provide means of determining the tensile strength of parachute silk and other fabrics under constant rate of loading conditions. The machine has a load capacity of 200 lb. From author's summary

2379. Kawada, T., On the plastic deformation of zinc bi-crystal. I, II, *J. phys. Soc. Japan* **6,** 5, 6; 362–366, 485–488, Sept./Oct., Nov./Dec. 1951.

The stress-strain curves of zinc bicrystal specimens consisting

of two single crystals adjoining each other were studied in tension tests. The shape of the cross section of the specimens changed sometimes into the elliptical form and sometimes into a heart- or wing-shaped form, according to the relative angles of slip and of the boundary plane with the tension axis. The observed stress-strain curve of the tensile stress vs. the unit elongation varied considerably for the various orientations of the bicrystals relative to each other. At fracture, all specimens were drawn down to wedge-shaped forms in the necked portion. The details of the mutual interaction of the two crystals are described and explained. A. Nadai, USA

2380. **Pfeil, L. B., High temperature materials: tests used as criteria of service behaviour,** *Schweiz. Arch.* **18,** 3, 88–97, Mar. 1952.

A general review of the significance from the design viewpoint of creep, fatigue, corrosion, and thermal shock properties of temperature-resistant materials. The significance of the onset of tertiary creep is discussed, and some new data are presented to show that creep beyond the tertiary stage reduces subsequent fatigue strength of Nimonic 80A. B. J. Lazan, USA

2381. **Lardner, E., and McGregor, N. B., Determination of elastic constants and stress/strain relationship to fracture of sintered tungsten carbide-cobalt alloys,** *J. Inst. Metals* **80,** 369–374, 1951–52.

Since an accurate knowledge of Young's modulus and Poisson's ratio is important in engineering design, an investigation has been carried out to determine these constants, as well as the stress/strain relationship to fracture, for sintered tungsten carbide-cobalt alloys containing 5–25% cobalt. Three alloys having in addition 5–15% titanium were also included. The data have been obtained by three different methods, none of which was capable of supplying completely the information required. The results show that, at room temperature, Young's modulus decreases considerably, and Poisson's ratio increases slightly, with rise in the cobalt content. Determinations of Young's modulus between room temperature and 600 C indicate a total decrease of the order of 5–6% at 600 C, the rate of decrease being almost uniform and independent of the amount of cobalt. There is evidence from the stress/strain relationship that when cobalt exceeds about 11%, some plastic deformation occurs at stresses well below the room-temperature fracture strength.

The effect of replacing some of the tungsten carbide with titanium carbide is further to lower Young's modulus. Variations in grain size between the limits investigated do not appear to cause any systematic change in Young's modulus or Poisson's ratio.

The energy to fracture in slow bending has also been determined and appears to be a more useful criterion of the service performance of sintered tungsten carbide-cobalt alloys than the bend strength or the impact resistance as measured by the usual methods.

From authors' summary by J. H. van der Veen, Holland

2382. **Mason, W. P., and McSkimin, H. J., Mechanical properties of polymers at ultrasonic frequencies,** *Bell. Syst. tech. J.* **31,** 1, 122–171, Jan. 1952.

Polymer molecules are capable of undergoing different types of motion, each of which has its characteristic frequency range. These motions can be (1) a configurational relaxation of the chain as a whole, (2) a change of position of a short segment of the chain molecule, and (3) a twisting of a short segment. These motions can be studied by measuring the response of the polymer material to mechanical or ultrasonic vibrations which are varied over a very wide frequency range. At least six types of instruments were used and are described for such measurements in solid polymers, rubbers, liquid polymers, and solutions. Considerable discussion and many data are given on the shear modulus and viscosity of polyisobutylene, polybutadiene, polypropylene, polypropylene sebacate, poly-α-methyl styrene, polystyrene, various rubbers, nylon, and polyethylene. The practical applications of such work lie in the fact that toughness, mechanical impact strength, and ultimate elongation depend upon the extent to which molecular motions can take place. Such measurements are also important in the selection of damping materials.

Lawrence Nielsen, USA

2383. **Schardin, H., Results of the kinematographic investigation of the glass fracture phenomenon** (in German), *Glastech. Berichte* **23,** 1, 3, 12; 1–10, 67–79, 325–336; Jan., Mar., Dec. 1950.

Author in 1939 found from cinematographic examination of the fracturing of glass, fracture velocities of 1500 m/sec for ordinary glass up to 2200 m/sec for quartz glass. This agrees with the work of Barstow and Edgerton [*J. Amer. ceram. Soc.* **22,** 302–307, 1939]. In the present work, there is studied the propagation of both elastic waves and fracture. The intersection of fracture fronts with elastic waves produces traces in the fractured area called Wallner lines. From these, if the velocity of the elastic wave is known, it is possible to calculate the fracture velocity. Three methods for making the elastic waves visible were used: (a) Double refraction produced by stress, (b) A schlieren-reflection method which gives sensitive visualization of the deformation of the glass surface during the fracture has the advantage of not being limited to transparent materials. It can be used, for example, on any ceramic where one surface can be ground and polished so that its optical quality is about equivalent to that of normal window glass. (*c*) Through sensitive adjustment of the schlieren the elastic waves may be made visible in transmitted light. Part I consists of the photographic details of these stress-optic and schlieren-optic methods. Part II has to do with the various types of waves in solid bodies and the relations between them; the origin of fracture and a discussion of primary and secondary fractures, the latter being all fractures which do not start directly from the center of the impact. In both parts I and II, rectangular glass plates of different thicknesses are fractured by an impact, and it is concluded that fracture always occurs due to a tensile stress; however, only a qualitative picture of the state of stress is drawn and this conclusion does not seem altogether justified to the reviewer. In part III, a study is made endeavoring to produce fracture by tearing, and the primary fracture takes place at the normal velocity of 1500 m/sec. A qualitative explanation of the bifurcation of a fracture crack is developed as well as a description of the formation of the Wallner lines. There is also a discussion of secondary fractures. In connection with this phenomenon it it interesting to compare Pugh, Heine-Geldern, Foner, and Mutschler [AMR **5,** Rev. 1749]. Edward Saibel, USA

2384. **Ahlberg, R., A contribution to the methods of measuring the plasticity of clays,** *Trans. Chalmers Univ. Technol.* no. 119, 24 pp., 1951.

A method of evaluating the plasticity characteristics of soils is presented that differs fundamentally from the usual Atterberg-Rieke method. Instead of determining the range of moisture content within which the soil displays a certain specified degree of plasticity, this method determines the comparative plasticity at various moisture contents.

The apparatus used is the Hoppler consistenciometer, a

cylindrical container in which the sample is placed and penetrated by a weighted sphere having a cross-sectional area of 2 sq cm. The velocity of fall of the sphere under various weights is plotted against pressure, giving a curve indicative of the plasticity characteristics of the soil.

Investigation of the cation complex as a factor influencing plasticity is also reported, but without conclusive results.

Reviewer believes that this method merits careful study as a possible tool of the civil engineer, although developed primarily for the use of the ceramics engineer.

Harold H. Munger, USA

2385. **MacCutcheon, E. M., and Wright, W. A., Transition characteristics of prestrained, notched steel specimens in tension,** *David W. Taylor Mod. Basin Rep.* **767,** 18 pp., Jan. 1952.

The prestraining of notched steel plates or structures above the brittle-to-ductile transition temperature was suggested as a means of increasing toughness in performance at lower temperatures. An exploratory study was made with edge-notched bars cut from $^3/_4$-in.-thick plates of four different steels. As a result of the prestrain procedure employed, three of the four steels exhibited somewhat greater toughness at a temperature just below the transition temperature.

An analysis of ship service data showed that small changes in toughness can influence the risk of fracture when marginal temperature conditions prevail. However, it was found that the lowering of the brittle-to-ductile transition temperature caused by the prestrain procedure was small, namely, about 5 F.

From authors' summary

2386. **Lubahn, J. D., Strain aging effects,** *Trans. Amer. Soc. Metals* **44,** 643–664, 1952.

Aging experiments were made with copper, aluminum, and mild steel. Aging near 400 C (750 F) causes both strengthening and the appearance of a yield point in specimens of 99.996% copper prestrained 2% at room temperature. 61S aluminum exhibits a yield point at −190 C (−310 F) when 1% strain at −190 C (−310 F) is followed by aging at room temperature. Analysis of published data reveals that there is a minimum in the rate sensitivity ($d \log S/d \log \epsilon$) of mild steel at about 250 C (480 F).

These facts, together with earlier facts from the literature, suggest that four mechanical effects—(a) strengthening of a strained metal during heating, (b) the appearance of a yield point in a strained metal after heating, (c) discontinuous flow, and (d) abnormally low rate sensitivity—occur together in many common metals and are most pronounced at the same temperature. Author concludes that the four effects are manifestations of the strain-aging process.

E. Siebel, Germany

Mechanics of Forming and Cutting

⊗2387. **Riegel, F., Calculations for metal cutting machine tools. Vol. I. Machining time, gear transmission calculations, taper cutting, thread cutting, dividing head work, relieving [Rechnen an spanabhebenden Werkzeugmaschinen. Erster Band: Hauptzeiten, Getriebeberechnungen, Kegelbearbeitung, Gewindeschneiden, Teilkopfarbeiten, Hinterdrehen], 3rd ed.,** Berlin, Springer-Verlag, 1951, x + 216 pp. DM 14.40.

Practical presentation of machining problems and their calculations. Formulas are given for turning, planing, grinding, milling, thread-cutting, boring and sawing, including formulas for the more difficult operations. Treatment of subject goes beyond the usual operator's handbook; sample calculations are correct within practical limits. The book will also be very helpful in laboratory and experimental shops with varied precision-machining tasks.

A. O. Schmidt, USA

2388. **Smith, C. L., Scott, F. H., and Sylwestrowicz, W., Pressure distribution between stock and rolls in hot and cold flat rolling,** *J. Iron Steel Inst.* **170,** part 4, 347–359, Apr. 1952.

The results of an investigation are described, in which the variation of pressure distribution over the entire area of contact between the rolled stock and rolls is studied for hot and cold-rolling, with and without back tension.

A photoelastic dynamometer is used in the experimental determination of the pressure distribution over the contact area. This dynamometer measures the normal force exerted on a small radial pin which is flush with the surface of the roll. Correction is made for the fact that the pin is of finite width.

Devices are described for applying and measuring back tension to the rolled strip and for measuring the total force separating the rolls. Results are given for the cold-rolling of annealed copper strip, 2 mm thick × 40 mm wide for reductions from 5 to 50%, and for similar hot-rolled copper strips with reductions of 18, 26, and 36%. These results are then compared with Dr. Orowan's analysis. The values of the yield stress of copper measured over the area of contact, as well as a method of measuring the coefficient of friction between the roll and rolled stock are also discussed.

Bernard W. Shaffer, USA

2389. **Kostron, H., Contribution to technological mechanics of deep drawing** (in German), Alfons Leon Gedenkschrift, Verlag Allg. Bau-Z., Wien = *Arch. Eisenhuttenw.* **22,** 7/8, 205–213, 1951. (See AMR **5,** Rev. 1433.)

2390. **Backer, W. R., Marshall, E. R., and Shaw, M. C., The size effect in metal cutting,** *Trans. ASME* **74,** 1, 61–71, Jan. 1952.

Shear stress for failure of steel SAE 1112 is estimated from results of grinding, micromilling (cutting minute chips at high speed), and tensile tests. Calculations show that theoretical shear strength of 1.8×10^6 lb per in.^{2} is attained for chip thicknesses below 30 μ in. Reviewer believes assumptions of force variation with depth of cut, extrapolation of shear energy (Fig. 14), and general indirectness of approach are open to criticism.

R. N. Arnold, Scotland

2391. **Outwater, J. O., and Shaw, M. C., Surface temperatures in grinding,** *Trans. ASME* **74,** 1, 73–81, Jan. 1952.

Due to high cutting speed and relatively small depth of cut associated with a grinding operation, both the surface temperature of the workpiece and the chip-grit interface temperature are much higher than those in a turning operation. Known methods for the calculation of temperature developed at sliding contacts are applied directly to estimate the surface temperature. When a low-carbon steel is ground, mean temperatures of over 2000 F are reported. The existence of such high surface temperature is held to be responsible for the presence of a thin transformed layer, which is largely retained austenite, beneath the ground surface.

Authors' analysis shows that when the depth of cut is less than the critical value to give theoretical shear strength of the work material, a lower surface temperature may be obtained by reducing the chip depth of cut. However, when it is above the critical value, the surface temperature may be reduced by increasing the chip depth of cut.

Wheel-work thermocouple technique is used to measure the average chip-grit interface temperature. It varies from 250 F

for tin to 6500 F for tungsten under the usual grinding conditions. Grinding atmosphere has been found to play an important role in the production of sparks as well as the energy required per unit volume of metal removal. Grinding forces may be increased 25-fold when air is replaced by nitrogen for the surrounding atmosphere.

B. T. Chao, USA

2392. **Marshall, E. R., and Shaw, M. C., Forces in dry surface grinding,** *Trans. ASME* **74,** 1, 51–58, Jan. 1952.

First of a series of three articles (see the two preceding reviews) which together ably develop the mechanics of grinding, using statistical and basic approach hitherto not done. In present paper, dynamometer is described for measuring, independently, normal and tangential forces in surface grinding, using bonded-wire strain gages. Effect of wheel grade, workmaterial hardness, depth of cut, table speed, wheel speed, work width and wheel-dressing techniques are explored for relatively mild grinding conditions. Brief x-ray diffraction study of depth of cold work and residual stresses is included. Specific energy is claimed to be the best criterion for interpreting data obtained. Specific energy was found to be very high compared to machining, and independent of workpiece hardness; it was also found to be independent of depth for small depths. Full significance of these and similar findings is apparent only when considered in connection with the findings of the second and third papers in the series.

M. Eugene Merchant, USA

2393. **Trent, E. M., Some factors affecting wear on cemented carbide tools,** *Machinery, Lond.* **79,** 2034, 2035; 823–828, 865–869; Nov. 1951.

Article presents a new theory of "cratering-wear" of carbide tools, especially when cutting steel. This wear is explained by a selective chemical attack of steel on tungsten-carbide grains under the influence of high local temperatures of about 1300 C generated by friction. Addition of titanium carbide decreases cratering because titanium-tungsten carbide has much higher resistance to chemical attack than tungsten carbide alone. Tungsten carbide-cobalt alloys give satisfactory performance when cutting nonferrous metals because these metals form no alloys with tungsten carbide which melt at a temperature appreciably below their melting points.

A probable heat distribution in the cutting tool is given which proves that temperatures up to 1325 C are possible at the center of the crater. The tool tip may be cooler than the crater center, which is appreciably further back up the tool face from the tip, by about 700 C. Consequently, high thermal conductivity of the tool material appears to be an advantage when cutting at high speeds and feeds since it reduces the temperature of the tool tip and hence the tendency of the tip to deform under the chip pressure. Addition of titanium carbide reduces thermal conductivity. Low thermal conductivity may be an advantage at lower cutting speeds and feeds since it reduces the tendency to build-up edge by keeping the tool hotter.

Author's findings are indeed very valuable in the further enlightenment of the metal-cutting problem.

Dimitri Kececioglu, USA

2394. **D'yachenko, P. E., Some results of the quality of surface of machine parts** (in Russian), *Izv. Akad. Nauk. SSSR Otd. tekh. Nauk*, no. 1, 22–28, Jan. 1951.

A general noncommittal discussion, expressing Russian leadership in the field, reports that new machines for testing surfaces have been designed in USSR, but cites only interferometers, double microscopes, and Profilometers as examples that are of German, British, or United States origin. Paper states that metal undergoes plastic deformation and elastic changes during the machining process, resulting in typical surface irregularities, and that the damaged surface will influence wear and life of parts. Author reiterates that manufacturers compete to improve surface quality and that improved testing methods must be found so that constructor can be certain of quality control. Article presents graphs showing results of differences in cutting speed, coolants, and lubricants on surface microgeometry for typical case. Reviewer believes paper presents little if any new information.

J. A. Broadston, USA

Hydraulics; Cavitation; Transport

(See also Revs. 2411, 2483)

2395. **Ramponi, F., Approximate determination of the turbulence in rectangular channels with small slope** (in Italian), *Energia elett.* **28,** 12, 695–696, Dec. 1951.

The method developed is applicable to wide rectangular channels of gradually varying depth, and is shown to give results in good agreement with those obtained by more involved procedures.

Glenn Murphy, USA

2396. **Craya, A., Evaluation of the critical regime in stratified flow,** *Trans. Amer. geophys. Un.* **32,** 6, 891–897, Dec. 1951.

Purpose of the present paper is to interpret the title's generalized concept [AMR **4,** Rev. 4025] from the hydraulics point of view. Consideration of the momentum principle and of the energy principle permits the formulation of two integral expressions for the cross section of a stratified current, the value of which can only decrease. These integrals nevertheless have a minimum value which is attained for a certain thickness of the current and for a certain velocity distribution, which by definition represent the critical regime. Critical regimes obtained by the two foregoing principles are not identical, though very nearly so.

Author selects two examples: A single layer of density $\rho_0 - \Delta\rho$ flowing over a motionless liquid of density ρ_0, and a continuous density distribution ranging from ρ_0 at the interface of the motionless liquid to $\rho_0 - \Delta\rho$ at the free surface. For both cases the phenomenon is governed by the parameter $\pi = (\Delta\rho/\rho_0)\, gh^3/q^2$, where q is the total discharge equally distributed over all the layers of a same density difference. The theory predicts only the thickness of the layers and does not determine the variation of the free surface. A brief discussion is given of a specific free-surface condition.

Paper is a condensation of that appearing in *Tellus* [AMR **4,** Rev. 3923] by same author.

From author's summary by H. G. Farmer, Jr., USA

2397. **Eisenberg, P., Effect of small errors in body shape and angle on pressure distribution and cavitation limits,** *David W. Taylor Mod. Basin Rep.* 792, 14 pp., Oct. 1951.

Shape considered is prolate ellipsoid. Shape error considered is a change in ellipsoid eccentricity. Shape error is related to error in minimum pressure coefficient. Angle error is related to most sensitive pressure coefficient. Both are referred to zero angle of attack. Results are extended to relation of errors to speed for inception of cavitation.

Robert B. Green, USA

2398. **Wuest, W., High pressure measurements with elastic measuring tubes** (in German), *ZVDI* **93,** 36, 1129–1131, Dec. 1951.

Because of unfavorable state of stress of tubular spring manometers, efforts were made to replace them by other devices using

(a) change of length or volume of straight tubes, under internal or external pressure; (b) bending straight tubes with originally eccentric bore; or (c) change of length or curvature of tubes with initial curvature. Description of these devices is given.

Alb. Schlag, Belgium

2399. Lagasse, J., and Nougaro, J., Electric device for measuring rapidly changing levels in hydraulics (in French), *C. R. Acad. Sci. Paris* **234,** 1, 56–58, Jan. 1952.

A familiar technique for measuring rapidly changing levels of a liquid is described. The measuring element consists of two parallel wires, immersed in the liquid, to which an a-c voltage is applied. The current variation in the electric circuit changes linearly with variations in liquid level.

Morris S. Macovsky, USA

2400. Escande, L., Amplitude of nondamped oscillations in surge tanks with throttling (in French), *C. R. Acad. Sci. Paris* **234,** 4, 405–408, Jan. 1952.

This paper concludes a previous note [AMR **5,** Rev. 1765] and gives complementary information and evidence on stability of oscillations in throttled surge tanks. The surge amplitude is given for the total and partial loading of the power station.

Ch. Jaeger, England

2401. Prigogine, I., On the surface tension of solutions of molecules of different sizes (in French), *J. Chim. Phys.* **47,** 1/2, 33–40, Jan./Feb. 1950.

Author calculates surface tension of binary solutions wherein some molecules are assumed to reside in a semicrystalline lattice and others are free to occupy two positions. The effect of the heat of solution on the surface tension is considered. The method, based on a statistical evaluation of the energy liberated, can be generalized for the case where one constituent is fixed and the other free to occupy a number of positions (r). The model used can be applied to organic solutions such as dibenzil in benzene or solutions of two hydrocarbons of different chain lengths.

From author's summary by B. R. Mead, USA

2402. Truesdell, C., On the viscosity of fluids according to the kinetic theory, *Z. Phys.* **131,** 3, 273–289, Feb. 1952.

Part I is a detailed analysis of the means by which various authors have approximated the relationship between λ and μ (the two viscosity coefficients introduced by Maxwell for gases). The various basic definitions of stress and temperature are carefully examined. It is concluded that no systematic rigorous kinetic theory of polyatomic gases yet exists.

Part II is a dimensional analysis of the dependence of viscosity on state and a discussion of the correct type of approximation formulas for gases and liquids.

W. L. Sibbitt, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 2353, 2354, 2433, 2436, 2448, 2479, 2517)

2403. Kuznetsov, D. S., Hydrodynamics [Gidrodinamika] (in Russian), Leningrad, Gidrometeoizdat, 1951, 391 pp.

A fine textbook for the Russian schools of hydrometeorology. The presentation is clear, vivid, and the reading a real pleasure, in reviewer's opinion.

The work is divided into six chapters and treats all fundamental questions as they occur in books on this subject. The first section deals with kinematics of perfect fluids. It explains such fundamental facts as the Euler and Lagrange theory, notion of the velocity potential, curl, stream function, etc.

The following main part is devoted to dynamics of perfect fluids. Along with the classical Euler and Lagrange investigations are given modern Gromeko, Fridman, and Joukowsky theories. Examples on usual kinds of flow illustrate theoretical considerations.

Chapters 3 and 4 deal with hydrostatics and with the study of "discontinuity surfaces," where functions characterizing the fluid motion become discontinuous. The pertaining facts are of importance for meteorology.

The next section develops theoretical foundations of the wave motions in perfect liquids and solves some important problems about this subject. According to its objectives, the book also treats such questions as theory of tide, waves in the rotating atmosphere, etc.

Dynamics of viscous fluids forms the subject of the last chapter.

Reviewer recommends the book to physicists and engineers. He considers it an excellent introduction to theoretical hydrodynamics.

V. Vodička, Czechoslovakia

2404. Imai, I., On Sneddon and Fulton's solution for the irrotational flow of a perfect fluid past two spheres, *J. phys. soc. Japan* **5,** 284–285, 1950.

Sneddon and Fulton [AMR **3,** Rev. 723] deal with the irrotational flow of a perfect incompressible fluid past two spheres by means of a formula of Weiss [*Proc. Camb. phil. Soc.* **40,** 259–261, 1944] for one sphere. The results of Sneddon and Fulton are a definite improvement over the results of Endo [*Proc. Phys.-Math. Soc. Japan* (3) **20,** 667–703, 1938] and Mitra [*Bull. Calcutta math. Soc.* **36,** 31–39, 1944] who have also considered the problem of two spheres. The author believes, however, that Weiss' formula is not applicable to two spheres as used by Sneddon and Fulton and, at best, is a good approximation to the two-sphere problem when the two bodies are far enough apart.

Courtesy of Mathematical Reviews A. Gelbart, USA

2405. Lapin, E., Crookshanks, R. J., and Hunter, H. F., Downwash behind a two-dimensional wing oscillating in plunging motion, *Douglas Aircr. Co. Rep.* SM-14198, 7 pp., 4 figs., Nov. 1951.

The integral for the downwash has been expressed in terms of Hankel functions and Schwartz functions. Asymptotic expansion shows that the downwash at large distances behind the wing varies sinusoidally with distance. Some graphs of the downwash as a function of the reduced frequency and the distance behind the wing are represented. The polar representation of the downwash at a given distance behind the wing is a divergent spiral, the modulus increasing asymptotically with the square root of the reduced frequency.

H. G. Loos, Holland

2406. Vallander, S. V., Calculation of flow around a cascade of airfoils (in Russian), *Doklady Akad. Nauk SSSR* (N.S.) **82,** 3, 345–348, Jan. 1952.

Author describes a method of conformal transformation for calculating airfoil cascades. He starts from the known transformation of the circle into a flat plate cascade. Further transformation from this plane into the airfoil cascade plane is expanded into Fourier series. Thus he gets the coordinates x, y of the airfoil

$$x = P\,(R, \beta_p, \vartheta) + A_0 + \textstyle\sum_{n=1}^{\infty}(A_n \cos n\vartheta - B_n \sin n\vartheta)$$

$$y = B_0 + \textstyle\sum_{n=1}^{\infty}(B_n \cos n\vartheta + A_n \sin n\vartheta)$$

where P is the known transformation function of a flat plate cascade, and ϑ is the angular coordinate on the circle. The unknown

coefficients A_n, B_n, and R, β_p are determined by an iterative process following S. G. Nuzhin (1947) and using as zero solution $A_n = B_n = 0$. Walter Wuest, Germany

2407. Stephenson, J. M., Secondary flow in cascades, *J. aero. Sci.* **18, 10,** 699–700, Oct. 1951.

Note is intended, first, to provide an alternate, simpler method of obtaining the results of Winter and Squire. As pointed out by Eichenberger [title source, 19, 2, Feb. 1952] Stephenson's assumptions are not consistent with the linearization of Helmholtz' equations made in the original paper. Stephenson's results cannot, therefore, be interpreted within the theory of vortex flow.

If the results of Winter and Squire are properly interpreted, no conceptual disagreement results between what Eichenberger and Stephenson say about the flow mixing at the trailing edge of a cascade. Winter and Squire showed that, if a plane irrotational flow is disturbed (the nature of the disturbance is not properly specified in the original paper or in the subsequent notes in the *J. aero. Sci.* but may be correctly stated to obtain the same results), the vorticity component along the streamlines (disturbance vorticity) may be calculated in terms of magnitudes defined by the basic irrotational flow and the nature of the disturbance upstream at infinity. The whole analysis is independent of the specific nature, i.e., of the boundary condition defining the basic flow. Hence, if the basic, irrotational flow corresponds to an infinite, two-dimensional passage, for the calculation of induced velocities due to the disturbance vorticity (secondary flow) *in a cascade* an additional vortex sheet should be assumed to account for the discontinuity in the flow originating in the difference in velocities on the outer and inner walls of the passage. If the basic flow is that corresponding to an airfoil cascade, then the disturbance vorticity uniquely determines the flow pattern as stated by Eichenberger.

For the calculation of that additional vorticity corresponding to the passage flow, the assumption made by Stephenson represents, at best, a rough approximation.

Fausto G. Gravalos, USA

2408. Kamimoto, G., On the conformal representation of circular wing lattice composed of airfoils with arbitrary shape (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17,** 60, 51–58, Oct. 1951.

A first-order theory of conformal representation of a given circular wing lattice into a circle is developed, assuming that the deviation of the airfoil shape from an arc of a logarithmic spiral is of small magnitude of the first order. An analytical expression for the velocity distribution on the airfoil is given. Numerical calculations are carried out for two practical examples, obtaining qualitative agreement with experimental results.

Isao Imai, Japan

2409. Kamimoto, G., On a method of conformal representation of a straight wing lattice composed of airfoils with arbitrary shape (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17,** 60, 58–63, Oct. 1951.

By means of the transformation function

$$z/c = \lambda/2\pi[e^{-i\beta}\log(k\zeta+1)/(k\zeta-1) + e^{i\beta}\log(\zeta+k)/(\zeta-k)]$$

a straight wing lattice in the z-plane is first conformally mapped on a nearly circular profile in the ζ-plane, which is then transformed into a true circle by the Theodorsen-Garrick method. Numerical calculations are carried out for a lattice of NACA 4412 airfoils with chord-pitch ratio $\lambda = 1.284$ and angle of stagger $\beta = 50°$. Isao Imai, Japan

2410. Graham, E. W., Disturbances arising from the instability of a jet, *Douglas Aircr. Co. Rep.* SM-14076, 18 pp., July 1951.

A simplified jet (or wake) is considered in a two-dimensional incompressible flow. This jet is unstable (assuming a small periodic deflection of wave length λ) because of the nature of the pressure distribution produced on a wave-shaped wall. Initially, the external fluid is undisturbed by the jet passing through it; however, as the jet distorts and continues its forward motion, the external fluid is disturbed and energy and momentum must be supplied to it. Since this energy and momentum can be provided only by the jet, the jet velocity is reduced from the initial value. This means that an increased amplitude of wake distortion is associated with reduced wake velocity. The normal velocity induced by the jet depends on the product of these two factors, and a maximum appears before the maximum deflection of the jet occurs.

From author's summary by S. A. Schaaf, USA

2411. Hudimoto, B., Kamimoto, G., and Yamamoto, T., On the cavitation characteristics of airfoils (in Japanese), *Trans. Japan Soc. mech. Engrs.* **7,** 60, 41–46, Oct. 1951.

A variant of the reviewer's method of calculating perfect fluid flow past an arbitrarily given airfoil [*J. aero. Soc. Japan* **9,** 88, p. 865, 1942] is presented, such that only the profile coordinates for standard points (0, 125, 2.5, . . ., 90, 100% chord points) are used, the drawing of the profile shape being unnecessary. Then, by the method, pressure distribution over four airfoils, NACA 23012, Munk 6, Clark Y, and Göttingen 429, are calculated at several angles of attack. Authors consider the minimum pressure point to be the starting point of cavitation, and give the position and the pressure coefficient there as functions of the angle of attack for each airfoil. The results of calculation are in good agreement with Numachi's experimental results. Isao Imai, Japan

2412. Kamimoto, G., Simizu, T., and Hirose, Y., Experiment on airfoils by the electrolyte tank method (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17,** 60, 47–50, Oct. 1951.

Streamlines of the flow fields with circulation around, and velocity distributions on the surface of a circular cylinder and a certain airfoil are determined by means of the electrolyte tank method. Lift coefficients of the airfoil are calculated for several values of angles of attack from observed values of circulation. The various experimental values are found to be in good agreement with the theoretical ones, thus showing the reliability of the electrolyte tank method. Isao Imai, Japan

2413. Davidson, P. M., The boundary conditions at a mist-water interface, *Brit. J. appl. Phys.* **2,** 7, 193–195, July 1951.

The boundary conditions at a mist-water interface for the differential equation of turbulent diffusion are formulated for two extreme cases, which may be described as the case of high turbulence and small droplets and the case of low turbulence and large droplets. In the former case (case 1), the boundary condition is found to be the same as for diffusion without mist. In the latter case (case 2), the boundary condition is stated to be the same as would be imposed by the presence of a laminar sublayer near the water surface, but has not been explicitly formulated.

Equations (6) and (7), pertaining, respectively, to cases 1 and 2, serve as the criteria to decide between these cases. The

validity of (6) vs. that of (7) under various physical conditions is the principal interest of experiments suggested by author (see following review).

The subject chosen is of great interest, and recognition of the two cases stated above shows penetration and originality. But reviewer believes that arguments lack rigor and the presentation lacks care. In Eq. (2), expressing the conditions of continuity of Y (corresponding to energy) and q_2 (total mass of water-vapor and liquid—per unit mass of atmosphere), the variations of these quantities in the wind direction have been omitted, and throughout the paper the heat transfer per unit area Q and the rate of condensation r at the wall have been treated as if they were constant. For this omission and this treatment, no reasons have been given, with all equations subsequent to (1) at stake. In spite of the frequent appearance of signs of partial differentiation with respect to the normal distance z, reviewer cannot suppress the impression that author considers Y and q_2 as functions of z above—among other things, the very lack of another coordinate substantiates this impression. This situation would correspond to one of no droplets and no heat and mass transfers (to the wall or within the fluid), and indeed to one of constant Y and vapor concentration all through the fluid right up to the wall—a trivial situation which is only asymptotically attained.

Chia-Shun Yih, France

2414. Evans, D. J., Experiments on the boundary conditions at a mist-water interface, *Brit. J. appl. Phys.* **2,** 7, 196–199, July 1951.

Experiments as suggested by Davidson (see preceding review) have been carried out over a wide range of physical conditions. Eq. (6) of Davidson (corresponding to case 1) is found to hold over a great range, while with diminishing turbulence the results tend to satisfy Eq. (7) of Davidson (corresponding to case 2).

Q and r are taken as the mean values of these quantities over the entire longitudinal distance under investigation, thus alleviating somewhat the difficulty concerning Q and r, but the difficulty concerning the longitudinal variations of Y and q_2 remains. Author states, in effect, that Q should be small (near the end of 2.1); thus he seems to realize that the longitudinal variations of Y and q_2 exist but should be kept small—an observation overlooked by Davidson. Nevertheless, further clarification concerning this point is needed to improve the theory of Davidson and to add to the weight of author's experimental results.

Chia-Shun Yih, France

2415. Hahnemann, H. W., On the heat-transfer coefficient in hydrodynamically and thermically "developed" laminar pipe flow (in German), *Forsch. Geb. Ing.-Wes.* (*B*) **18,** 1, 25–26, 1952.

The temperature distribution has been calculated in a fluid which shows a laminar flow inside a straight tube of circular cross section. Only the stationary state, neglecting end effects, has been considered. For the temperature, the following boundary conditions are introduced: The wall temperature shows a linear gradient, while the incoming fluid has a temperature differing from that of the wall at the mouth of the tube. A heat-transfer coefficient is defined and calculated.

H. C. Brinkman, Indonesia

2416. Carletti, L., Viscosity effect on transient waves in narrow fuel lines (in German), ETH, Zürich, Thesis no. 1984, 65 pp., 1951.

From the one-dimensional, unsteady, viscous equations of motion and continuity, author derives solutions for the pressure and velocity due to an initial pressure wave in an infinite line. The viscous effects are assumed to be laminar with a parabolic profile. Solution gives an exponentially decaying pressure or velocity moving undistorted with sonic speed. Theory does not predict an experimental phenomenon of a secondary trailing wave. This is discussed in detail. The experimental setup is described as consisting of a large closed circular line with provisions for introducing a pressure impulse and measuring the resulting pressure waves. Agreement between theory and experiment is discussed.

H. P. Liepman, USA

Compressible Flow, Gas Dynamics

(See also Revs. 2439, 2448, 2452, 2471, 2476, 2477, 2486)

2417. Miles, J. W., On nonsteady supersonic flow about pointed bodies of revolution, *J. aero. Sci.* **19,** 3, 208–209, Mar. 1952.

Note in Readers' Forum.

2418. Pope, A., Aerodynamics of supersonic flight, New York, Toronto, London, Pitman Publ. Corp., 1950. xi + 180 pp. $4.

As stated in the preface, this book is intended to provide "an elementary one-term course which aims to introduce a student to the field of supersonics and hence help him to decide whether or not he would like to go ahead with a more advanced course at a higher level." Prerequisites are differential and integral calculus and one term of elementary aerodynamics. Book has seven chapters: (1) Fundamental relations; (2) Flow in a duct; (3) Two-dimensional flow; (4) The oblique shock; (5) Approximate theories; (6) Supersonic wind tunnels; and (7) Three-dimensional flow. Each chapter contains examples, worked out in numerical detail, which illustrate principles and the use of derived equations. Several problems are provided at the end of each chapter.

Reviewer found the quality of the book to be very uneven. A total of more than 30 statements and passages appear which reviewer considers to be of questionable accuracy or poorly written. The net effect was an unfavorable one with regard to the value of the book. However, several features were noted which, to some extent, counteract this impression. These include the excellent selection and organization of material and the maintenance of an appropriately elementary discussion which avoids thermodynamical and mathematical concepts that may be unfamiliar to the undergraduate engineer. Physical reasoning rather than mathematical deduction is frequently emphasized to advantage in explanation of basic supersonic flow phenomena. The usefulness of a book that introduces principles and concepts of supersonic flow to engineers with little background in aerodynamics and mathematics is fairly evident. This book could be recommended without reservation to fill this need if the numerous questionable passages were rewritten or eliminated.

W. E. Moeckel, USA

2419. Goldsworthy, F. A., Supersonic flow over thin symmetrical wings with given surface pressure distribution, *Aero. Quart.* **3,** part 4, 263–279, Feb. 1952.

The inverse problem of determining the supersonic flow past a thin symmetrical wing at zero incidence with given surface pressure distribution is solved for wings of arbitrary planform. Expressions are obtained for the vertical derivative of the potential on the wing surface from which, using the linearized boundary condition of zero normal velocity at the surface, the profile of the wing can be designed. The integral-equation method adopted by J. C. Evvard and extended by G. N. Ward

is used. The analysis cannot be applied to pointed wings, whose leading edges are subsonic. The results in part I are completely general and are applied to specific problems in part II.

From author's summary by Pierre Schwaar, Switzerland

2420. **Evvard, J. C., and Maslen, S. H., Three-dimensional supersonic nozzles and inlets of arbitrary exit cross section,** *NACA TN* **2688, 12 pp.,** Apr. 1952.

A method is presented for obtaining three-dimensional unsymmetric supersonic nozzles and inlets from known axisymmetric flows. Streamlines bounding the desired exit shape are traced through the known basic flow solution to give the required unsymmetric wall contours. Several examples are given.

From authors' summary by Angelo Miele, Argentina

2421. **Cabannes, H., Study of the attached shock wave in axisymmetric flow** (in French), *Rech. aéro.* no. 24, 17–23, Nov.–Dec. 1951.

Tip region of an ogival obstacle is considered. Equations of motion in spherical coordinates are solved by means of following series:

$$u(r, \theta) = u_0(\theta) + \frac{r}{R} u_1(\theta) \ldots$$

$$v(r, \theta) = v_0(\theta) + \frac{r}{R} v_1(\theta) \ldots$$

and similarly for $\rho(r, \theta)$ and $p(r, \theta)$. This results in the usual equations of conical flow of zero order for functions of index 0, which may be integrated numerically by reference to flow conditions on obstacle. Functions of index 1 are determined by reference to flow conditions on shock wave. Numerical results are given for an obstacle tip angle of 40° and 6 values $1.2 < M < 1.9$. Values of quotient of curvature radii of shock wave and obstacle at the tip are also presented for same M-range.

Reviewer does not agree with author's assertion that results concerning tip curvature of shock wave are exact, since using same adiabatic constant in whole fluid space after curved shock wave, as he does, results in an approximation which, of course, is not detrimental for all practical cases of slender bodies at moderate M-values.

Case of obstacle with zero tip curvature may be treated by means of following series:

$$u(r, \theta) = u_0(\theta) + 2Ar^2 \cdot u_2(\theta) + \ldots$$

etc., since in previous series, functions of index 1 would be multiplied with zero coefficients. Essentially the same calculation procedure is outlined. Finally, some values of quotient of tip-curvature radii according to Lin are given as comparison.

Reviewer believes that practical application of results will be confined to the obstacle tip region, where greatest errors of linear theory occur.

Pierre Schwaar, Switzerland

2422. **Oppenheim, A. K., A contribution to the theory of the development and stability of detonation in gases,** *J. appl. Mech.* **19,** 1, 63–71, Mar. 1952.

See AMR **5,** Rev. 786.

2423. **Rao, G. V. R., Two-dimensional subsonic flow past elliptic cylinder by the variational method,** *NACA TN* 2666, 39 pp., Mar. 1952.

Direct variational method due to Hargreaves and Bateman is applied to two-dimensional compressible flow past elliptic cylinders. Boundary-value problem and corresponding variational problem are formulated in elliptic coordinates, and the Rayleigh-Ritz method is used to obtain close approximation. Difficulties arising from infinity of considered domain are investigated, and corrections resulting therefrom are introduced. Numerical examples are given for several thickness ratios and Mach numbers. Good results are obtained for flow past thick body at low Mach number and for flow past thin body at high Mach number.

A. van Heemert, Holland

2424. **Okazaki, T., Two-dimensional subsonic flow past wing lattice** (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17, 60,** 78–81, Oct. 1951.

Two-dimensional subsonic flow past a wing lattice is considered on the basis of Prandtl-Glauert's linearized theory, assuming that the unperturbed stream is the vector mean of the velocity fields infinitely upstream and downstream of the lattice. It is concluded that the zero-lift angle and lift slope of the lattice in a compressible flow are $(1 - M^2)^{-1/2}$ times those for a modified lattice with pitch smaller, by a factor of $(1 - M^2)^{1/2}$, than the original one in an incompressible flow, where M is the unperturbed stream Mach number.

Isao Imai, Japan

2425. **Bryson, A. E., Jr., An experimental investigation of transonic flow past two-dimensional wedge and circular-arc sections using a Mach-Zehnder interferometer,** *NACA TN* 2560, 97 pp., Nov. 1951.

Interferometer measurements are given of the flow fields near two-dimensional wedge and circular-arc sections at zero angle of attack at transonic velocities. Both partially supersonic flows and flows with detached shock waves are investigated, and pressure distributions and drag coefficients vs. M are obtained. The wedge data are compared with the theoretical works of various authors.

A linearization of the shock relations for the case of transonic flow is developed which justifies smooth fairing of data obtained in the high subsonic and low supersonic velocities through sonic velocity.

From the author's summary by Martin D. Schwartz, USA

2426. **Hansen, A. G., and Martin, M. H., Some geometrical properties of plane flows,** *Proc. Camb. phil. Soc.* **47,** part 4, 763–776, Oct. 1951.

This paper is of fundamental importance in the plane-flow-analysis field. Starting with a geometric configuration consisting of tangent to the streamline, lines of constant pressure, temperature, velocity and density at a point in the physical plane, many new and interesting theorems are proved. The first of these is: "The tangents drawn to an isobar in the physical plane and to a streamline in the hodograph plane at corresponding points are perpendicular to each other."

It is refreshing to see a concise paper as general and important as this one presented in a clear and precise manner.

Keeve M. Siegel, USA

2427. **Cooper, M., and Hamilton, C. V., Orientation of orifices on bodies of revolution for determination of stream static pressure at supersonic speeds,** *NACA TN* 2592, 26 pp., Jan. 1952.

Experimental pressure-distribution data are given for a parabolic body of revolution of high fineness ratio, and comparison is made with slender-body theory for various angles of incidence. Results obtained at $M = 1.59$, $R = 3.6 \times 10^6$ show locations at which surface pressure orifices give readings equal to the free-stream pressure. The range of incidence at which a single orifice reads within 1% of stream static pressure is determined.

The range of pitch and yaw obtainable with similar precision for the common reading of two symmetrically disposed orifices is computed from the above results. It is suggested that this method be used on bodies of revolution for free-stream static pressure measurement at supersonic speeds.

H. M. Spivack, USA

2428. Glass, I. I., An experimental determination of the speed of sound in gases from the head of a rarefaction wave, *Inst. Aerophys. Univ. Toronto UTIA Rep.* 9, 34 pp., Dec. 1951.

An alternative definition of sound velocity is the rate of propagation of the head of a rarefaction wave into gas at rest, as it follows from nonlinear Riemann shock-tube theory.

A camera is equipped to yield a continuous representation of a longitudinal window in the tube, i.e., a portion of the (x,t) plane. By means of schlieren technique the finite density gradient at the head of a rarefaction wave is visualized; the sound velocity so measured is in good agreement with $(\gamma RT)^{1/2}$ and acoustic results.

Experimental equipment and some records are presented.

Gino Moretti, Argentina

2429. Legras, J., Conic flow in the neighborhood of a junction point (in French), *C. R. Acad. Sci. Paris* **234,** 2, 181–183, Jan. 1952.

Author deals with conic supersonic flows around a rectangular plate; the attacked border is perpendicular to the direction of the undisturbed flow. There are plane shock waves (envelopes of the shock cones starting from the attacked border) and conic shock waves issuing from the vertices of the right angles of the plate. These plane shocks and conic shocks meet along two junction lines. By generalizing a method due to G. B. Witham [AMR **4,** Revs. 1245, 1248], author introduces a parametric representation of the potential function holding in the neighborhood of the junction points. By this representation one can determine the shock wave and compute the velocities between the shock and the obstacle.

Robert Sauer, Germany

Turbulence, Boundary Layer, etc.

(See also Revs. 2353, 2482, 2500, 2534)

2430. Batchelor, G. K., Turbulent diffusion (Lectures, no. 4, prepared by S. I. Pai), Univ. Maryland, Inst. Fluid Dynamics Appl. Math., 10 pp., Apr. 1951. $.40.

Lecture, covering part of material in *Australian J. sci. Res.* (A), **2,** 437–450, 1949, considers mechanism of diffusion in the simple case of diffusion of a cloud of marked fluid particles about average position of their center in a homogeneous field of turbulence. It reduces to the problem of statistical behavior of a single fluid particle. Theory shows dispersion increases first as square of time of flight t, then more slowly, and eventually increases linearly with t. Several different experiments have shown probability distribution of displacement is normal for all values of t. Thus, diffusion can be represented by differential equation of heat-conduction type, diffusion coefficient initially increasing with t and eventually constant.

From author's summary by Hugh L. Dryden, USA

2431. Batchelor, G. K., Diffusion in a field of homogeneous turbulence. II. The relative motion of particles, *Proc. Camb. phil. Soc.* **48,** part 2, 345–362, Apr. 1952.

This article concerns the relative diffusion of particles in a marked cloud in a turbulent fluid, in which the turbulence is assumed infinite in extent and spatially homogeneous. In effect, this is a study of the tendency of such a cloud of fluid to change its shape. The aspect of relative diffusion presented is determined from the initial shape of the cloud and the statistical properties of the diffusion of two particles of given initial separation. Relative diffusion is an accelerating process until the particles are far enough apart to wander independently. The hypothesis is made that if the initial separation is small enough, the probability distribution of the separation will tend asymptotically to form independent of the initial separation, before the particles move independently. A discussion of the representation of relative diffusion by a differential equation is presented, various forms being suggested for specific conditions.

E. G. Allen, USA

2432. Ribner, H. S., and Tucker, M., Spectrum of turbulence in a contracting stream, *NACA TN* 2606, 54 pp., Jan. 1952.

Paper discusses theoretical attempt to predict the effect on turbulence of a contracting stream, for example, in the entrance of a wind tunnel. Assuming weak turbulence and an inviscid fluid, author uses spectrum tensor to predict behavior of turbulence. The compressibility of the main stream is considered, but that connected with turbulence is not. General method of approach is given in detail but, for results to be explicit, the spectrum before contraction must be given. For the case of axisymmetric contraction and isotropic turbulence with an experimentally determined spectrum, the spectra of velocity components are computed and results given in curves. The effect of viscosity as shown by the decay of the turbulence is evaluated by a rough approximation. Comparison is made with experimental results for longitudinal and lateral intensity of turbulence, but agreement is not consistent. Curves for use of wind-tunnel designers are given, but reviewer believes that these will not be accepted until further experimental verification is obtained. With the equipment available at present, this should not be difficult to do.

W. D. Baines, Canada

2433. Lin, T. C., and Schaaf, S. A., Effect of slip on flow near a stagnation point and in a boundary layer, *NACA TN* 2568, 28 pp., Dec. 1951.

Consideration is given to the usual incompressible viscous flow equations for flat-plate boundary-layer and stagnation-point flows, but where the boundary conditions are now taken to be those for slip-flow of rarefied gases. By accounting for the slip effects in the boundary conditions only, it is found that the stagnation pressure is increased by the slip effect, and that there is only a negligible effect on the flat-plate skin-friction coefficient in the restricted range of applicability of this analysis.

Simon Ostrach, USA

2434. Howarth, L., The boundary layer in three-dimensional flow. Part II. The flow near a stagnation point, *Phil. Mag.* (7) **335,** 1433–1440, Dec. 1951.

Using the general equations given in part I [AMR, **4,** Rev. 3640], author discusses boundary-layer flow in the vicinity of a stagnation point on a general (three-dimensional) surface. The velocity components of the potential flow outside the boundary layer are $U = ax$, $V = by$ (x, y are coordinates along the surface). Equations can be reduced to a pair of simultaneous ordinary third-order differential equations containing the single parameter $c = a/b$ which varies from 0 (corresponding to two-dimensional flow) to 1 (corresponding to axial flow past a body of revolution). If ζ is the distance from the surface and $z = \zeta(a/\nu)^{1/2}$ (ν is kinematic viscosity), the velocity components in the boundary layer are $u = axf'(z)$, $v = byg'(z)$, $w = -(\nu/a)^{1/2}[af(z) + bg(z)]$ where $f'^2 - ff'' - cgf'' = 1 + f'''$, $g'^2 - gg'' - 1/cfg'' = 1 + 1/cg'''$. The solutions f, g, f', and g' of these equa-

tions are given in tables for the values $c = 0, 0.25, 0.50, 0.75$, and 1. When c is small, it is convenient to have expansion $f = f_0 + cf_1 + c^2f_1 + \ldots$ and $g = g_0 + cg_1 + c^2g_2 - \ldots$ The values of $f_0', f_1', f_2', g_0', g_1', g_2'$ are also given in a table. At a point x, y, the direction of the skin friction is inclined to the main flow direction at an angle

$$\tan^{-1}\frac{g''(0)}{f''(0)}\frac{cy}{x} - \tan^{-1}\frac{cy}{x}$$

The boundary-layer thickness is different for the two principal directions of flow components, but they do not differ vastly. Both thicknesses decrease as c increases.

Alb. Betz, Germany

2435. **Wuest, W., Development of a laminar boundary layer behind a suction point,** *NACA TM* 1336, 18 pp., Mar. 1952.

See AMR **4**, Rev. 1692.

2436. **Michel, R., Determination of the transition point and calculation of the drag of wings in incompressible flow** (in French), *Rech. aéro.* no. 24, 41–48, Nov.–Dec. 1951.

The abscissa s_T parallel to the surface of T, point of maximal value of u'/U (U is velocity over the surface of two-dimensional potential flow, u' root mean square of turbulence fluctuations in U-direction) on a line parallel to the surface and normal to the tales, turned out to be independent of the ordinate of T (reviewer found that s_T only varied 3% chord with the ordinate on a NACA 0018 profile at $\alpha = 0°$ and $Re = 1.2$ and 2.6×10^6). Comparison of calculated drag as a function of the place of the transition point Tr with measured drag showed that $s_{T_r} = s_T$.

The points $(U\theta/\nu)_T$ vs. $(Us/\nu)_T$ for a number of airfoils at various flow conditions (θ momentum thickness, ν kinematic viscosity, suffix T indicates values at T) turned out to lay on one single curve, depending only on tunnel turbulence. The same was true for J_T vs. $(Us/\nu)_T$; $J = (1/U^6s)_0\int^s U^6 ds$. So, intersection of J-curve and J_T-curve gives the place of the point that has to be used as transition point for drag calculations.

H. Wijker, Holland

2437. **Schröder, K., A simple numerical method for the calculation of the laminar boundary layer,** *NACA TM* 1317, 47 pp., Apr. 1952.

Translation of *ZWB Forschungsber.* 1741, Feb. 1943.

2438. **Tifford, A. N., On certain particular solutions of the laminar boundary-layer equations,** Proc. Midwest. Conf. Fluid Dynamics, 1st Conf., May 1950; J. W. Edwards, Ann Arbor, Mich., 81–90, 1951. $10.

Because of the relative simplicity of the boundary-layer equations, when applied to a surface along which the local velocity outside the boundary layer varies as a power function of the distance from the stagnation point, Falkner and Skan's family of so-called "similar" boundary-layer profiles have been widely used in theoretical studies of the properties of laminar boundary layers. Author searches for other families of similar profiles which might offer additional aid in such studies. Investigating the differential equation for the flow in the boundary layer, he finds two possibilities for the boundary layer to give similar profiles. In these the velocity distribution in the boundary layer does not vary along the surface. In one case, the external velocity distribution obeys an exponential law; in the other case, it is a power function. In both cases, the boundary-layer flow fulfills a nonlinear differential Eq. (10). The physical significance of the new boundary-layer flows is discussed.

Reviewer finds the calculations to be correct. Author seems not to find it worth while to mention two other possibilities, viz., in case IV, $p = 2 + 3c$; $a = 1 - K_1(1 + 2c)/2$; $n = 1/2 - 3K_1(1 + 2c)/2$. $c_1 = 0$ gives, in case I, another connection between the constants. Neither of these flows fulfills Eq. (10).

O. H. Faxén, Sweden

2439. **Ferrari, C., Velocity and temperature distribution through the laminar boundary layer in supersonic flow,** *J. aero. Sci.* **19**, 1, 39–47, 65, Jan. 1952.

Paper presents method for treating the practically interesting problem of laminar compressible boundary layers with axial pressure gradients and heat transfer. Usual boundary-layer assumptions are made of (a) constant pressure through the layer, (b) constant wall temperature, (c) constant Prandtl number, and (d) polynomial temperature-viscosity relation. Transformation of von Mises is applied to both the momentum and energy equations. The Kármán-Millikan technique is then employed to obtain "external" and "internal" solutions to both transformed equations; these are joined. For the case of heat transfer with a prescribed wall temperature, a successive approximation scheme is developed to simplify calculations. Estimate of the influence of the boundary layer on the pressure gradient given by potential flow considerations is obtained by computing normal velocity component and resultant potential flow deflection at edge of boundary layer, and by applying linearized potential flow relations. The results of applying method to the case of an adverse pressure gradient are given briefly.

Boundary-layer specialists interested in details will want to refer to the more extensive report of the method and applications available through Johns Hopkins University, Applied Physics Laboratory. Reviewer notes that method predicts recovery factor equal to Prandtl number, in variance with experiment.

Paul A. Libby, USA

2440. **Ringleb, F. O., Computation of the laminar boundary layer with suction,** *J. aero. Sci.* **19**, 1, 48–54, Jan. 1952.

The incompressible laminar boundary layer is computed, using as approximate velocity profile

$$u = U\{1 - \exp[a(x)y + b(x)y^2 + c(x)y^3 + d(x)y^4]\}$$

If the velocity outside the boundary layer and the suction velocity are given functions of x, the coefficient $a(x)$ is determined by an ordinary differential equation of the first order. The other three coefficients are determined by elementary functions of the first coefficient and its first derivative. The method is applied to some examples with known solution and shows high accuracy, although the boundary-layer equation is only fulfilled at the wall. Reviewer states that the same approximation has been used by A. Betz [*Arch. Math.*, Karlsruhe, **2**, 220–222, 1949/50]. Unpublished computations have shown that the above method gives very good results in the case of similar profiles, but that in more general cases the approximation is no better than other known methods.

W. Wuest, Germany

2441. **Mager, A., and Hansen, A. G., Laminar boundary layer over flat plate in a flow having circular streamlines,** *NACA TN* 2658, 28 pp., Feb. 1952.

Aimed at better understanding of secondary flows in turbomachinery, analytical investigation was made of development of laminar boundary layer on semi-infinite flat plate placed in plane flow with concentric circular streamlines and small total turning angle. Shape of velocity profiles along the perpendicular to mainstream flow is presented for incompressible flow and for compressible gas flow with Prandtl number unity.

For incompressible flow, boundary layer in main flow direction is identical with Blasius solution. Deviation angle of boundary-

layer flow at plate surface from local main-stream direction is toward center of curvature and is 3.26 times the turning angle of the main stream.

For compressible flow, both boundary-layer thickness and deviation angle increase with Mach number.

Ascher H. Shapiro, USA

2442. Hudimoto, B., Approximate methods of calculating boundary layers (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17,** 60, 93–96, Oct. 1951.

Velocity distribution across the laminary boundary layer is expressed as a family of curves involving two parameters, one of which is the usual Pohlhausen parameter and the other represents the effect of d^2U/dx^2, where U is the outside velocity and x is the distance along the surface. The second parameter is selected so that the velocity distribution at separation point agrees approximately with that of exact solution for the case $U = U_0 - U_n x^n$ [$n = 1$: Howarth, *Proc. roy. Soc. Lond.* (*A*) **164,** 1938; $n = 2, 4, 8$: Tani, AMR **4,** Rev. 3320]. The first parameter is then determined by satisfying the momentum relation of the boundary layer. No numerical example is given, illustrating the application of the method to the general case of velocity gradient. A short comment is appended, suggesting the existence of a similar effect of d^2U/dx^2 on the separation of turbulent boundary layer.

Itiro Tani, Japan

2443. Hudimoto, B., On the diffusion of wake of a body (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17,** 60, 96–98, Oct. 1951.

Author extends theory of two-dimensional wake based on Prandtl's mixing length postulate to the case where the outside flow slightly accelerates or decelerates. It is found that the velocity distribution across the wake is sensibly the same as the case of uniform flow, whereas the breadth of wake increases more rapidly when the outside flow decelerates as compared to the case of uniform flow. The theoretical results agree fairly well with the measurements made downstream of a circular cylinder placed in a convergent or divergent wind channel of rectangular section.

Itiro Tani, Japan

2444. Burrows, D. L., and Schwartzberg, M. A., Experimental investigation of an NACA 64A010 airfoil section with 41 suction slots on each surface for control of laminar boundary layer, *NACA TN* 2644, 37 pp., Apr. 1952.

An investigation has been made of boundary-layer suction through flush surface slots as a means for increasing the extent of laminar flow on the NACA 64A010 airfoil section. The 3-ft-chord model was designed according to an analysis presented herein to maintain nearly full-chord laminar flow at Reynolds numbers up to 25×10^6 with the use of 41 suction slots on each surface.

Laminar flow was maintained over at least 0.91 chord on one surface up to a Reynolds number of 10×10^6. A like extent of laminar flow on the other surface would have resulted in a net drag saving of about 50% over the plain smooth airfoil at Reynolds numbers as high as 10×10^6. This result was obtained only after the expenditure of a great amount of effort in forming slot-entry contours that would not cause transition, and in maintaining the surfaces of the model at the edges of the slots sufficiently smooth. Extensive laminar flow was not obtained at higher Reynolds numbers because of the increasing sensitivity of the flow to minute surface irregularities and slight inaccuracies of slot-entry contour.

From authors' summary

2445. Bidwell, J. M., Application of the von Kármán momentum theorem to turbulent boundary layers, *NACA TN* 2571, 25 pp., Dec. 1951.

Von Kármán momentum theorem is derived for incompressible turbulent flow including terms containing fluctuating velocity components. Author concludes that streamwise derivative of turbulent longitudinal momentum $\overline{\rho u'u'}$ may be of sufficient magnitude to require its inclusion in the application of the von Kármán momentum theorem to turbulent boundary layers near separation.

Morris W. Rubesin, USA

2446. Zaat, J. A., The calculation of the point of turbulent-boundary-layer separation on an NACA 0018 profile and the determination of the optimum angle of incidence, *Nat. LuchtLab. Amsterdam Rap.* F.89, 14 pp., 7 tables, 9 figs., Sept. 1951.

Boundary-layer calculation methods recently proposed by the author [AMR **4,** Rev. 4546] are applied to a thick airfoil. Since no boundary-layer measurements over this airfoil are available, the methods cannot be evaluated directly. However, the calculated chordwise variations of the turbulent-boundary-layer shape parameter are very different from those measured on comparable airfoils, which tends to cast doubt on the accuracy of the methods.

Gerald Nitzberg, USA

2447. Corrsin, S., Heat transfer in isotropic turbulence, *J. appl. Phys.* **23,** 1, 113–118, Jan. 1952.

Author constructs a theory of heat transfer for nondecaying isotropic turbulence in incompressible flows. It is postulated that the mean temperature gradient is constant over any region that is large compared with any characteristic dimension of the fluctuations. The development is patterned after Taylor's theory of diffusion by continuous movements. Under the assumptions, formulas for turbulent heat-transfer coefficient and heat-transfer correlation coefficient in terms of velocity fields and physical properties of the fluid are derived. Naturally, in following the Taylor analysis, the formulas are derived by both Lagrangian and Eulerian analyses.

Paper also contains a derivation of an equation for temperature fluctuations and a preliminary result for turbulent Prandtl numbers in shear flow.

Several comparisons with experimental results are made; however, as the author notes, definitive critiques are impossible at present due to the familiar reasons: experimental scatter, nonrealization of assumed conditions, compressibility, turbulent decay, molecular conduction, etc.

Knox Millsaps, USA

2448. Corrsin, S., and Kovasznay, L. S. G., The energy equation for two kinds of "incompressible flow," *J. aero. Sci.* **18,** 12, 843–844, Dec. 1951.

Following a comment by Krzywoblocki, authors consider whether c_v or c_p should be used in the energy equation for an "incompressible" fluid. For a truly incompressible liquid, c_v should be used, but for motion of a gas at low Mach number, the choice may depend on the type of flow. Authors point out that in cases in which the range of pressure variation is a small fraction of the absolute pressure, c_p is the more accurate choice.

G. K. Batchelor, England

2449. Bass, J., General equations of space-time correlations in a turbulent fluid (in French), *C. R. Acad. Sci. Paris* 234, 8, 806–808, Feb. 1952.

Author supposes that stationary homogeneous turbulence can exist, and derives the dynamical relation between the double and triple isotropic velocity correlations for two points in space time.

G. K. Batchelor, England

Aerodynamics of Flight; Wind Forces

(See also Revs. 2405, 2444, 2478)

2450. **Schuldenfrei, M., Comisarow, P., and Goodson, K. W., Stability and control characteristics of a complete airplane model having a wing with quarter-chord line swept back 40°, aspect ratio 2.50, and taper ratio 0.42,** *NACA TN* 2482, 86 pp., Dec. 1951.

Report presents results of tests to determine the low-speed longitudinal and directional characteristics of model of swept-wing airplane. Raising the horizontal tail to a position near the top of the vertical tail improved the longitudinal stability at the higher lift coefficients. The directional stability at small angles of sideslip was improved by flattening the top of the dorsal and by revising the vertical tail-fuselage intersection. Aileron effectiveness data are presented and the effects of geometric dihedral, of wing-end plates, and of dorsal fin modifications are included. Robert M. Crane, USA

2451. **Raikh, A., Calculation of the lateral-dynamic stability of aircraft,** *NACA TM* 1264, 82 pp., Feb. 1952.

Translation from *Trudy Tsentral. Aero-Gidrodinam. Inst.* no. 453, 1939.

2452. **Robinson, A., and Davies, F. T., The effect of the sweepback of delta wings on the performance of an aircraft at supersonic speeds,** *Aero. Res. Counc. Lond. Rep. Mem.* 2476, 6 pp., Mar. 1947, published 1951.

Paper presents results of calculations of the effects of sweepback or aspect ratio on the total drag of triangular-wing aircraft at supersonic speeds. It concludes that large angle of sweepback is not uniformly beneficial, especially at high altitudes. This is primarily because the induced drag is large for delta wings having large sweepback (low aspect ratio). Author disposes of trailing-edge sweep as "relatively irrelevant," a conclusion not confirmed by conical wing theory (see e.g., Puckett, A. E., and Stewart, H. J., AMR **1**, Rev. 151). Failure to include trim drag also limits usefulness of calculations. Experimental data do not confirm the discontinuities in drag predicted by conical flow theory when the Mach number normal to the leading edge attains a value of unity. Robert M. Crane, USA

2453. **Hayward, L. H., A review of helicopter patents,** *Aircr. Engng.* **24**, 278, 92–105, Apr. 1952.

2454. **Strong, J. G., Star-tracking missiles,** *Aeroplane*, 212–215, Aug. 24, 1951.

Paper is a speculative article on the possibilities of using automatic star-tracking systems for the guidance of offensive long-range missiles. Material presented was purportedly inspired by a disclosure that the United States is making progress on the development of such a system. The conception of a celestial sphere and the elements of star navigation are discussed. Also, some highly speculative remarks about the nature of possible tracking mechanisms are set forth. Details considered in these discussions, however, tend to belie author's prefacing comment that the question of guiding long-range offensive missiles seems to have been overlooked until recently.

Arthur L. Jones, USA

2455. **Press, H., and McDougal, R. L., The gust and gust-load experience of a twin-engine low-altitude transport airplane in operation on a northern transcontinental route,** *NACA TN* 2663, 33 pp., Apr. 1952.

An analysis of the gust and gust-load experience of a low-altitude transcontinental transport airplane, based on 834 hours of VGH record, is presented. The relation of gust experience to airspeed, altitude, season of the year, and flight condition is considered. An estimate of the over-all gust history for the present type of operations is derived from a synthesis of the VGH data and available V-G data. From authors' summary

2456. **Wisniewski, M. L., Longitudinal stability fundamentals,** *Aircr. Engng.* **23**, 274, 362–367, Dec. 1951.

Study presents basic concepts of the static longitudinal stability of an airplane. The contributions of wing, fuselage, nacelles, and horizontal tail surface are dealt with for the stick-fixed condition. The effects of power on equilibrium and stability for both propeller and jet aircraft are considered. Formulas for numerical computation of the different contributions to the stability are included. J. Buhrman, Holland

2457. **Gates, O. B., Jr., A theoretical analysis of the effect of several auxiliary damping devices on the lateral stability and controllability of a high-speed aircraft,** *NACA TN* 2565, 39 pp., Dec. 1951.

A theoretical study is made of the effect of auxiliary damping devices on lateral stability and controllability of a high-speed aircraft. Auxiliary damping is introduced through a device which deflects the rudder or auxiliary surface in proportion to yawing velocity, rolling velocity, or rolling acceleration, or through one which deflects both aileron and rudder proportional to rolling velocity. No attempt is made to assess the effect of either control system or aerodynamic lag on the performance of the airplane with dampers. The calculations show that each of the stabilization systems can improve the damping of the lateral oscillation. The system where the rudder was deflected in proportion to rolling acceleration resulted in large adverse yaws after aileron deflection. All methods resulted in larger rudder deflections to maintain zero sideslip subsequent to aileron deflection when compared with the airplane without auxiliary damping. The damper proportional to yawing velocity required an increase in rudder deflection to obtain a steady rate of yaw and roll angle when compared with the basic system.

John E. Stevens, USA

2458. **Nowakowski, W., and Sandauer, J., Lateral stability of aircraft** (in Polish), *Techn. Lotn.* **7**, 1, 4–11, Jan./Feb. 1952.

In the first part [AMR **5**, Rev. 1844], authors derived and discussed general equations of motion. In the present part, authors derive formulas for values of derivatives, so important in stability calculations. Derivation is performed under the assumption that derivatives are sums of factors appearing on wing, fuselage, and tail surfaces. Briefly, problem is treated as linear, and interference is ignored. Authors do not discuss the conditions under which this assumption is valid. Reviewer presumes it is valid for planes of small wing-loading at moderate subsonic velocities. A supplementary discussion on this subject would be valuable. Authors take into account various planforms of wings, V-form of wings, moderate sweptback wings, etc. The calculations are very simple, and the mathematics used is elementary. Formulas are given to recalculate derivatives with respect to principal axes of inertia. Numerical example for a glider with the use of some wind-tunnel tests closes the paper which has value for designers of gliders and light planes.

M. Z. Krzywoblocki, USA

2459. **Decker, J. L., Horizontal tail loads in abrupt pull-ups from level flight,** *J. aero. Sci.* **18**, 12, 835–841, Dec. 1951.

Paper represents still another approach to the application of

the so-called "inverse" method [see AMR **4**, 4444; **5**, 845]. A time history of the airplane normal load factor, in the form of a power series, is assumed, and the time history of horizontal tail load subsequently determined. The usual assumptions of constant forward speed and steady flow are made. The only arbitrary parameter in the method is the maximum rate of elevator deflection. It is tentatively correlated with the airplane limit-load factor and the flight speed, and is expressed as a simple function of these two variables.

Gabriel Isakson, USA

2460. Timman, R., and Lemaigre, B., The lifting line of arbitrary form considered as the limit case of the lifting surface in incompressible flow (in French), *Nat. LuchtLab. Amsterdam Rap.* F.95, 19 pp., Sept. 1951.

An integral equation for a lifting surface is developed from the vortex sheet point of view. The wing planform is then assumed to be of high aspect ratio with a midchord line of finite curvature. With the aspect ratio defined to be of order $1/\epsilon$, the double integrals which appear in the equation are expanded in powers of ϵ, and terms of order ϵ or higher are neglected. This procedure leads to a relatively simple integral equation for a lifting line of finite curvature. Integral equations for an unswept rectangular wing, a yawed wing, and a wing of small midchord line curvature are exhibited as special cases. It is difficult to assess the practical utility of the results obtained, inasmuch as methods for solving the integral equation are not presented. From the mathematical point of view, the development by a consistent procedure of a lifting-line-type integral equation which differs from the Prandtl theory for unswept wings is of interest.

Herbert R. Lawrence, USA

2461. Neel, C. B., Jr., and Steinmetz, C. P., The calculated and measured performance characteristics of a heated-wire liquid-water-content meter for measuring icing severity, *NACA TN* 2615, 59 pp., Jan. 1952.

A type of icing-rate meter is proposed. The variation in resistance with temperature for a wire loop being known, the heat transfer by convection may be calculated and the energy used in evaporation deduced from the measured altitude, air temperature, air speed, voltage, and current. Good agreement between measured and predicted performance was obtained up to liquid water contents where the wire was incapable of evaporating all the impinging moisture before blow-off. Original data and analyses are compared.

Myron Tribus, USA

2462. Walker, P. B., The experimental approach to aircraft structural research, *J. aero. Sci.* **19**, 3, 145–160, Mar. 1952.

Research in aircraft structures is constantly bringing forward new problems of a fundamental character. Many of these respond to systematic experiment, and the paper presents the experimental approach in terms of recent work done in Great Britain and particularly at the Royal Aircraft Establishment.

One aspect of research considered is the testing of small specimens made of xylonite. Another is the strength testing of actual aircraft, which opens up several fields of research activity for discussion. Design studies of new testing equipment for large aircraft and for pressure cabins are also described. Consideration is given to the measurement of external forces in flight and the design of measuring instruments, including a description of the new counting accelerometer recently developed by the R.A.E. Finally, structural fatigue is discussed as being one of the most difficult subjects with which the structural specialist has yet had to deal.

From author's summary

2463. Graham, E. W., and Rodriguez, A. M., The characteristics of fuel motion which affect airplane dynamics, *Douglas Aircr. Co. Rep.* SM-14212, 52 pp., Nov. 1951.

The response of the fuel in a rectangular tank to harmonic motions of three different types, viz., (a) horizontal translation parallel to a tank wall, (b) pitching about a horizontal axis parallel to a tank wall, and (c) yawing about a vertical axis, is studied. The fluid is assumed incompressible and nonviscous. Expressions are given for the velocity potential satisfying the Laplace equations as well as the linearized boundary conditions at the tank walls (prescribed normal velocity) and at the free surface (constant pressure). Linearization is allowed if the angular displacements of the tank are small, if the accelerations are small compared to the acceleration of gravity, and if the forcing frequency is not too near to one of the frequencies of free oscillation of the fuel. Forces and moments exerted by the fluid on the tank walls are calculated. The reactions due to motions arbitrary in time are obtained by aid of Laplace transforms.

Mechanical systems consisting of springs and masses and exerting the same total forces and moments on the tank as the fuel are derived; viz., one system for motions (a) and (b), provided the pitching axis is perpendicular to the direction of translation, and another system for motion (c). Introduction of these equivalent mechanical systems, of which the numerical characteristics can be taken from diagrams in report, will be useful in dynamic analyses.

A. I. van de Vooren, Holland

Aeroelasticity (Flutter, Divergence, etc.)

2464. MacNeal, R. H., McCann, G. D., and Wilts, C. H., The solution of aeroelastic problems by means of electrical analogies, *J. aero. Sci.* **18**, 12, 777–789, Dec. 1951.

Paper describes computer used at California Institute of Technology for solution of problems in field of "dynamics of deformable airplane."

Previous analog computers have focused attention on the differential equation of the system. An electrical circuit is constructed with the same equations as the mechanical systems to be studied, and with proportional constants. Voltages and currents in network may be interpreted as values of the analogous mechanical variables.

In the Caltech computer, the differential equations do not enter explicitly, and, in fact, need not be formulated. The physical system (in general with continuous distribution of mass, stiffness, and applied air loads) is replaced by a finite number of segments. The inertial properties of each segment and the loads on it are represented by appropriate electrical components. The electrical coupling of each segment to adjacent ones reproduces the stiffness properties of the system. The analog of voltage is velocity; of current, force; of inductance, mass; etc.

Disturbance voltages may be applied to the network at a number of points to represent transient or steady-state forcing functions, and values of current and voltage throughout the network can be converted to velocities and forces. This permits, for example, the determination of normal modes in complex wing-body-tail combinations; and the determination of wing-bending moments, shears, and torques in gust loading.

In the electrical network, the identity of the various parts of the airplane is retained. Thus it is relatively easy to make design studies leading to optimum configurations.

Bernard Etkin, Canada

2465. Runyan, H. L., Cunningham, H. J., and Watkins, C. E., Single degree of freedom flutter of an aileron, *J. aero. Sci.* 19, 3, 215–216, Mar. 1952.

Referring to paper by Abichandani and Rosenberg [AMR **5**, Rev. 500], authors show that the flutter frequency should be greater than the aileron natural frequency for sufficiently small distance between aileron leading edge and hinge axis.

From authors' summary

2466. Runyan, H. L., Effect of various parameters including Mach number on the single-degree-of-freedom flutter of a control surface in potential flow, *NACA TN* 2551, 33 pp., Dec. 1951.

Theoretical calculations on basis of two-dimensional potential flow are made of the possibility of single-degree-of-freedom control-surface flutter. The effects of structural damping, aerodynamic balance, axis of rotation, and compressibility are considered. Calculations indicate the existence of single-degree-of-freedom flutter of a control surface.

Henry G. Lew, USA

2467. van de Vooren, A. I., Theory and practice of flutter calculations for systems with many degrees of freedom, Thesis, Leiden, Eduardo Ijdo N.V., 1952, 102 pp.

Paper gives a very thorough account of some methods of handling flutter problems when many degrees of freedom are involved; author states that ten degrees of freedom will be necessary in the near future. However, only methods that apply generalized coordinates are treated in the paper, while reviewer believes the forced-vibration method of analysis may be advantageous in many such cases. The aerodynamic forces are all derived from the usual two-dimensional incompressible flow theory, and integrations over the span are performed as if each strip were part of an infinite airfoil. Main contribution of the paper lies in the further development of the Galerkin and iteration methods as applied to flutter calculations.

N. O. Myklestad, USA

Propellers, Fans, Turbines, Pumps, etc.

2468. Crigler, J. L., and Gilman, J., Jr., Calculation of aerodynamic forces on a propeller in pitch or yaw, *NACA TN* 2585, 33 pp., Jan. 1952.

Report investigates by analytical and experimental means the applicability of conventional aerodynamic analysis of a propeller for the cases of yaw or pitch. It is shown that the aerodynamic loading in yaw or pitch can be represented satisfactorily by steady-state strip analyses at various blade positions occurring during one revolution.

Material of this report has since been substantiated and expanded by more recent work in the propeller industry and the NACA.

James B. Duke, USA

2469. Brown, W. B., and Bradshaw, G. R., Design and performance of family of diffusing scrolls with mixed-flow impeller and vaneless diffuser, *NACA Rep.* no. 936, 10 pp., 1949.

A comparison of over-all performance was made on a compressor which consisted of an available 11.24-in. diam mixed-flow impeller, a 20-in. diam vaneless diffuser, and a family of five scrolls of different cross-sectional shapes. Diffusing scrolls were specially designed on the basis of a constant static pressure at the scroll entrance, and the circumferential location of a succession of preselected cross-sectional areas was determined by considering the radial variation in fluid density and the effects of friction along the scroll. Purpose was to establish influence of scroll shapes (divergence angles of 24°, 40°, and 80° were tried) and possibility of improvements over original compressor design with vaned diffuser and collector ring. Range of test was 700 to 1300 fps tip speed, from full throttle to surge.

Comparison showed that change in scroll geometry had negligible influence on performance, but substitution of scroll for collector ring gave increases in efficiency of 3 to 12 points, substantial increases in pressure coefficient, and extended the operating range to significantly higher values of load coefficient.

Otmar E. Teichmann, USA

2470. Mortarino, C., Experiments on a cascade of blades for compressors (in Italian), *Aerotecnica* **30**, 2, 59–72, 1950.

Experiments were conducted (a) to define the more suitable devices and procedures for determination of characteristics of an infinite cascade of blades as a base for design of axial compressors; (b) to compare experimental and theoretical results; (c) to obtain appropriate data for design of noses, energy losses, influence of boundary layer, and clearance between blades and walls in the case of fixed ones.

Velocities of 60 m/s (196 ft/sec), Reynolds numbers of 400,000, and Mach numbers of 0.17 were attained. Hence, it was assumed that compressibility had no effect.

The blades (NACA 6512) were made of wood, but one was metallic with several holes connected to a manometer to measure surface pressures. Velocities were determined with a spherical Pitot and a single 1-mm micro-Pitot. Wind tunnel had a test section of 0.35 × 0.35 m (14 in. × 14 in.) and was actioned by a Leonard, 75-kw and 3000-rpm direct-current group.

Cascade was operated at several inclinations with respect to axis of test section in order to obtain several incidences.

Author applied his method of determining direction of flow, using small droplets of oil paint on the surface of blades, which move with the stream and describe a path made visible by the color. Experiments showed flow was far from plane motion. Forces against airfoil were evaluated from pressure diagrams obtained.

Author compares his own results with those of Ferrari's theory.

Reviewer believes it would be interesting, in view of the uncertainties of application of these results to real problems of compressors, to experiment with a sector cascade of radial blades, with an angle between each one. It is believed this is not very difficult to do with the same test section of the wind tunnel.

Armando Balloffet, Argentina

2471. Briggs, W. B., Effect of Mach number on the flow and application of compressibility corrections in a two-dimensional subsonic transonic compressor cascade having varied porous-wall suction at the blade tips, *NACA TN* 2649, 43 pp., Mar. 1952.

Author describes tests of cascade of compressor blades of NACA 65-(12)10 section over a range of inlet Mach number from 0.12 to 0.89. By use of a boundary-layer suction slot on each of the side walls ahead of the cascade as well as suction through porous sections of the walls at the blade tips, something approaching two-dimensional flow was simulated, the criterion being that the two-dimensional continuity relation before and after the cascade should be satisfied. Experimental data include variations with Mach number for a given geometric setting of the blades of turning angle, wake, pressure distributions, and static pressure rise. These data were obtained both with and without the boundary-layer control. An attempt is made to see whether the variation of pressure distribution with Mach

number can be correlated with the Prandtl-Glauert rule, the Kármán-Tsien rule, and a third rule based on what the author describes as the vector mean velocity contraction coefficient. None of these results shows consistently close agreement with experiment, although for certain over-all requirements the agreement in some instances might be acceptable.

A. D. Young, England

2472. Davidson, I. M., Some data pertaining to the supersonic axial-flow compressor, *Aero. Res. Counc. Lond. Rep. Mem.* 2554, 36 pp., May 1947, published 1951.

Part I (See AMR **4**, Rev. 3614) is a brief presentation of properties of shocks significant for design of two-dimensional transonic compressor cascades, including experimental results. Tentative design rules are stated. Part II is a qualitative discussion of possible supersonic compressor performance. Part III is a short summary of early German work [Weise, Encke, and Betz] demonstrating possibility of satisfactory supersonic axial-flow compressor operation, although mechanical failures limited scope of investigations.

Richard G. Folsom, USA

2473. Stephenson, J. M., A solution of the surging problem in axial-flow compressors, *J. aero. Sci.* **19**, 1, 67–69, Jan. 1952.

Article deals with the problem of matching stages in a multistage compressor and, in particular, with the control of the matching by the use of cooling during the process of compression. In the analysis it is shown that, if an axial-flow compressor has many stages on a single rotor shaft and adiabatic flow, there is only one combination of pressure ratio and mass flow for which the efficiency of compression is a maximum. At other operating conditions, the efficiency is reduced and surging may be encountered. Because of these effects, author suggests an upper limit of about six on the design pressure ratio of such a compressor. When cooling is used during the compression, the analysis indicates that this upper limit may be raised to more than 10. Brief comments on the methods of applying this idea and on other methods of overcoming the surge problem are included.

Wm. R. Hawthorne, England

2474. Voaden, G. H., Index testing of hydraulic turbines, *Trans. ASME* **73**, 5, 481–488, July 1951.

Author recommends index-testing of reaction turbines as a means for determining the gate opening and load at which peak efficiency is obtained and the relative efficiencies at other gate openings and loads. On adjustable-blade propeller turbines, the optimum blade-angle gate-opening relationship can be established in the field and the blade-gate cam can be corrected accordingly. Among the methods of determining the relative discharge, the Winter-Kennedy method (utilizing radial pressure difference in turbine spiral) and the J. A. Peck method (two piezometers in one of the stay vanes between case and wicket gate) are discussed in detail, and a complete example of an index test on an adjustable-blade propeller turbine is given. Since the flow rate determination gives only relative values, the efficiencies have to be interpreted in terms of a known value, such as per the manufacturer's "experience curve." Except for measurement of discharge, index tests may be conducted in accordance with the ASME Power Test Code and are useful where absolute methods are considered inadvisable because of some unusual condition or because of expenses involved.

Otmar E. Teichmann, USA

2475. Rütschi, K., Investigations of volute pumps of various speeds (in German), *Schweiz. Arch.* **17**, 2, 33–46, Feb. 1951.

An experimental analysis is made covering a complete line of single-stage volute pumps. Tests include data on about 30 units covering a complete range of specific speeds. Test data show the usual performance characteristics of head, horsepower, and efficiency as functions of flow capacity. In addition, the impeller of each group of pumps is reduced from its maximum diameter to a value of 80% of the maximum diameter, showing the effects on the performance characteristics. An analysis of the cavitation characteristics is also presented. Paper presents a complete analysis of pumps over a very wide range of both specific speeds and sizes.

The test data are used to analyze similarity laws relating to change of efficiency with size and Reynolds number. For three specific speeds the test data are compared with theories of Moody, Pfleiderer, Staufer, and others. Author concludes that none of these theories represents his test results correctly. There is no simple relationship between efficiency and Reynolds number. He believes that the change of efficiency as function of impeller-inlet diameter is the best means of showing the effect of size.

H. E. Sheets, USA

Flow and Flight Test Techniques

(See also Revs. 2425, 2427, 2432, 2464)

2476. Gontier, G., Experimental determination of the transonic border in plane flow (in French), *C. R. Acad. Sci. Paris* 234, 4, 403–405, Jan. 1952.

Theoretically, a "transonic border" may exist in the supersonic portion of an accelerating mixed flow field such that disturbances produced downstream of this border modify the subsonic flow upstream (cf., e.g., Germain, AMR **5**, Rev. 1808). Present note describes experimental method of determining transonic border in throat of a small two-dimensional tunnel. Results show agreement with referenced theoretical determination.

J. S. Isenberg, USA

2477. Corrsin, S., Effect of wind-tunnel nozzle on steady-flow nonuniformities, *J. aero. Sci.* **19**, 2, 135–136, Feb. 1952.

Analytical relations are derived for determining: (a) The effect of upstream density irregularities on the working-section velocity irregularities in a low-speed tunnel; and (b) the effect, in compressible flow, of upstream irregularities in both mean velocity and stagnation temperature. It is assumed that the flow is isentropic in the nozzle, and that the static pressure is constant across the duct both before and after the nozzle. One numerical example showed that the temperature irregularity caused much more trouble than the upstream velocity irregularity.

R. C. Binder, USA

2478. Mandl, P., and Pounder, J. R., Wind tunnel interference on rolling moment of a rotating wing, *Nat. Res. Counc. Canada* AR-10, 31 pp., 1951.

See AMR **3**, Rev. 2764.

2479. Sanders, J., and Pounder, J. R., Wall interference in wind tunnels of closed rectangular section: development of equations, *Nat. Res. Counc. Canada* AR-11, 26 pp., 1951.

Report is mathematical supplement to paper AR-7 ("Wall interference in wind tunnels of closed rectangular section"). Assuming lifting-line theory, the expressions for wall-induced velocity at points along wing span are derived for symmetrical airfoils with arbitrary lift distribution. Calculations are extended to unsymmetrical airfoils and airfoils placed above or below tunnel center line. Expressions are also obtained for

induced velocity and downwash correction at points downstream of the wing. Effects of compressibility are not considered.

H. Julian Allen, USA

2480. **Jaques, C. N., Instrumentation for testing aircraft and aero-engines,** *J. roy. aero. Soc.* **55,** 492, 762–781, Dec. 1951.

Paper outlines importance of basic principles of dynamic measurements upon accuracy, such as the influence of the damping ratio upon magnification factor and phase angle.

Discussing accelerometers, it is shown that, for measurement of unknown transients, a system of high natural frequency pickups followed by electronic amplification is the best solution. Unbonded resistance pickups are better than the bonded type; inductance pickups are, however, advisable for highest sensitivity. An RAE counting accelerometer consisting of a spring-mass system coupled to rotary mass-spring system with an endless belt is discussed. Damping is provided by eddy currents.

Paper deals further with an electronic engine-speed instrumentation, using a fork or crystal time-interval generator and an electronic counter. The device, which exists also in an airborne version, has errors less than 10 rpm in a range of 400 to 20,000 rpm. An engine torquemeter for airborne use measures the phase angle between two generators on the shaft, consisting of toothed wheels and stationary coils. The phase angle is proportional to the distortion and is measured by an electronic device.

A fuel flowmeter (Negretti and Zambra) is based on a constant pressure-difference orifice operating an electrical transmitter. Constant pressure difference is realized by means of a bypass device acting on a piston, producing linear relationship between piston travel and mass flow. Device is temperature-compensated, insensitive to turbulence, and has accuracy of $\pm 1\%$.

Dealing further with strain recorders, author mentions several devices for 50, 96, and 600 ways, partly equipped with automatically operated switches. Continuously tracing multichannel recorders for signal frequencies up to 2000 cps are further discussed.

H. L. Studer, Switzerland

2481. **Hall, J. G., Flow structure and pressure recovery in a supersonic open jet wind tunnel,** *Nat. aero. Establ. Canad.* LR-19, 17 pp., 2 tables, 20 figs., Nov. 1951.

Flow structure and pressure recovery in the N.R.C. open-jet wind tunnel have been investigated as functions of supersonic diffuser throat area at Mach numbers 1.4, 1.8, and 3.5 with the tunnel empty.

Balance chamber pressure is independent of supersonic diffuser throat area unless the diffuser functions as a second supersonic nozzle with subsonic flow in the diffuser entrance, sonic flow at the throat, and supersonic downstream of the throat. Smooth control of balance chamber pressure by adjustment of the diffuser throat is obtained only at low Mach numbers.

At low Mach numbers, below about 1.8 in the present case, maximum pressure recovery for model testing is limited to that for a parallel jet, the diffuser functioning as a second supersonic nozzle. In this case, the recovered pressure ratio is closely given by the ratio of supersonic nozzle to diffuser throat areas. At high Mach numbers, a pressure recovery exceeding that for a parallel jet with sonic flow at the diffuser throat may be obtained, the test rhombus flow being suitable for model testing. This more efficient diffusion at high Mach numbers is achieved by starting the tunnel with the diffuser throat fully open and closing it up after supersonic flow has developed throughout. In this case, the flow remains supersonic in the diffuser entrance.

From author's summary

2482. **Benthem, J. P., Turbulence investigation by measurement of temperature fluctuations of a hot wire placed in the airstream** (in Dutch), *Nat. LuchtLab. Amsterdam Rap.* A.1198, 69 pp., tables, figs., Apr. 1950.

Report gives a detailed treatment of the response of a hot-wire anemometer, its static and dynamic calibrations, and of various measurements in a turbulent flow.

A nomogram permits estimation of the time lag of a hot wire. Nonlinear effects are discussed.

R. Betchov, USA

2483. **De Troyer, A., van Itterbeek, A., and van den Berg, G. J., Measurement of the viscosity of liquid helium by means of the oscillating disc method,** *Physica* 17, 1, 50–62, Jan. 1951.

The purpose of this work was to determine the viscosity of the liquid helium and to note if there was any discontinuity as the temperature passed through the λ-point. Measurements were made by means of an oscillating disk. Results were corrected for thickness of oscillating disk and for the influence of the surrounding liquid. It was found that there was no discontinuity in the viscosity as the temperature passed through the λ-point.

Glen N. Cox, USA

2484. **Holder, D. W., and North, R. J., Colour in the windtunnel,** *Aeroplane,* 16–19, Jan. 4, 1952.

Article reviews the schlieren and shadow optical techniques commonly used in flow observations about models in high-speed wind tunnels. A number of illustrations are given. The title is misleading in that color is only incidental. A colored image may be had by using a prism but, as remarked by author, this is rarely done in scientific investigations. Reviewer believes that the article contains no new information.

G. B. Schubauer, USA

2485. **de Lathouder, A., Methods for wind-tunnel measurements of models of vehicles, vessels, and constructions,** *Nat. LuchtLab. Amsterdam Rap.* A.1241, 5 pp., 1951.

It is not possible to give a clear-cut scheme which is generally applicable to measuring methods for wind-tunnel research on models of vehicles, vessels, constructions, etc. The choice of the mounting depends among other things on the answer to the following questions: (a) Does the object on full scale move with respect to the supporting plane? (b) Does the object touch the supporting plane in some points only (car: slit flow), or does it make full contact with this plane (ship: no slit)? (c) Are there certain advantages in measuring the base plate together with the model (model forces not too small with respect to the aerodynamic forces acting on the plate, a number of models or model configurations)? (d) Has a high accuracy to be pursued?

A general view is given of a number of useful methods, classified according to the conditions mentioned under (a) and (b). The advantages and disadvantages of the various methods are considered. At the same time, points (c) and (d) are raised so far as these might be of any importance.

From author's summary

2486. **Drougge, G., Note on wall interference in two-dimensional flow at subsonic and transonic speeds,** *Flygtekn. Försöksanst. Medd.* 40, 14 pp., 1951. Kr 1.50.

Author gives theoretical fundamentals of extrapolation method for determining tunnel-wall interference effects in subsonic and transonic two-dimensional flows. By means of Tsien-Lees' formula, giving distribution of the Mach number M along the profile chord when profile is put between two walls in linearized subsonic flow, author writes, for a symmetrical parabolic profile, relations among: Mach number M_∞ of the free stream, M_{max}

maximum Mach number along the chord, and the distance $2h$ between the walls. A numerical calculation shows that the M distribution is about the same if M_∞ for every h is chosen so as to give the same M_{max}; it follows that $C_D M_\infty$ = const, under above conditions, where C_D is the wave-drag coefficient. Author shows that, having experimental diagrams C_D as function of M_∞ for two similar profiles corresponding to two different values of h/l (l = chord-length), the values of C_D and M_∞ corresponding to $h = \infty$ for the same profile can be obtained by the extrapolation method. Reviewer believes that results obtained and proposed by author are not valid for transonic flows. Carlo Ferrari, Italy

2487. Stever, H. G., and Rathbun, K. C., Theoretical and experimental investigation of condensation of air in hypersonic wind tunnels, *NACA TN* 2559, 79 pp., Dec. 1951.

Experiments performed in a small hypersonic tunnel at the Massachusetts Institute of Technology showed that condensation of air components occurred if the static temperature of the air fell more than a few degrees below equilibrium saturation values. The possibility that the observed condensation was due to the presence of nuclei of water, carbon dioxide, or other impurities was not, however, eliminated. A modification of the nucleation theory of condensation is presented which takes into account a postulated variation of surface tension with nucleus size. Resulting condensation rates are higher, for a given supersaturation condition, than those predicted by unmodified theory, and indicate that spontaneous condensation of oxygen and nitrogen can take place at relatively small supersaturation even if condensation initiated by foreign nuclei is negligible.

W. E. Moeckel, USA

Thermodynamics

(See also Revs. 2402, 2487)

2488. Hercus, E. O., Elements of thermodynamics and statistical mechanics, New York, Cambridge Univ. Press; Melbourne at the Univ. Press, Austral., 1950, ix + 153 pp. \$3.75.

This text is written for undergraduate students as a first introduction to thermodynamics. The selection of topics and their presentation have been abridged (72 pp., thermodynamics; 44 pp., statistics; 23 pp., appendix) to a point where it is hardly possible for the reader, on the basis of knowledge conveyed in this book, to analyze and solve simple problems in the subject fields, or even to understand the basic thermodynamic principles. The book may be used with profit, however, as an incentive to collateral study of more extensive texts.

Eric F. Lype, USA

2489. Gardner, Willard H., Gardner, Walter H., and Gardner, W., Thermodynamics and soil moisture, *Soil science* **72**, 2, 101–105, Aug. 1951.

Authors treat a problem arising in classical theory of thermodynamics, which they state is of importance in the application of thermodynamics to studies on soil moisture. The problem in question is connected with the definition of the chemical or partial potential equations (Gibbs notation)

$$\mu_i = (\partial\epsilon/\partial m_i)_{\eta,v,m_k} = (\partial\zeta/\partial m_i)_{p,t,m_k}$$

Authors propose an alternative way of defining μ_i, from which they conclude that

$$\mu_i = (\partial\zeta/\partial m_i)_{p,t,m_k} = (\partial\epsilon/\partial m_i)_{p,t,m_k}$$

In reviewer's opinion, the starting point of authors expressed in their equation (f), viz., $d\epsilon = td\eta - pdv$, where η and v are considered as functions of p, t and m_i, is erroneous. This can easily be seen by considering the case where $\epsilon = m_1\epsilon_1 - m_2\epsilon_2$, while analogous equations hold for the other extensive variables. On the other hand, Gibbs' definition need not lead to ambiguities. Authors also doubt the generally accepted constancy of ζ for reversible processes at constant temperature and pressure. Reviewer points out that in their example the difficulty can be removed by noticing that in the right-hand side of Eq. (1) the term $d\epsilon = (\epsilon_2 - \epsilon_1)dm$ is lacking.

D. A. de Vries, Holland

2490. Jones, W. M., Gordon, J., and Long, E. A., The heat capacities of uranium, uranium trioxide, and uranium dioxide from 15°K to 300°K, *J. chem. Phys.* **20**, 4, 695–699, Apr. 1952.

Heat-capacity measurements have been made over the temperature range 15–300 K for uranium metal, uranium trioxide, and uranium dioxide. A maximum was observed at 28.7 K in the heat-capacity curve of UO_2, which is probably the result of the changing population of the magnetic quadrivalent uranium ions among their available energy states. The entropies of these substances at 298.16 K are 12.03, 23.57, and 18.63 cal deg^{-1} $mole^{-1}$, respectively. The free energies of formation of UO_3 and UO_2 are -273.1 ± 3, and -246.6 ± 0.6 kcal $mole^{-1}$, respectively.

From authors' summary

2491. Uchida, H., Mercury vapor tables and $i - s$ diagram (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17**, 62, 70–77, Nov. 1951.

Thermodynamic tables and enthalpy-entropy diagram of saturated and superheated mercury vapor are presented for the pressure from 0.001 to 60 kg/cm² and the temperature below 700 C. For this purpose, author determines characteristic equations such as saturation pressure-temperature relation and specific heat-temperature relation for liquid mercury, using the experimental data of various authors, while for the specific volume of saturated liquid mercury the formula of Hoffman and Meissner is used.

Humio Tamaki, Japan

2492. Lewis, B., and von Elbe, G., Combustion, flames and explosion of gases, New York, Academic Press Inc., 1951, xix + 795 pp. \$13.50.

This up-to-date account of science and technology relating to combustion of gases by two of the leading investigators in the field forms a welcome and much needed reference work on the subject. The book should be intelligible to the reader with elementary knowledge of physical chemistry and thermodynamics. It is well printed and contains extensive references to current literature. Sufficient emphasis has been placed on the possibility of alternate mechanisms of combustion and different interpretations of experimental data to enable the reader to draw his own conclusions on controversial subjects. In this respect, the current edition is superior to the authors' previous versions.

The discussion of technical combustion processes is relatively short compared to the treatments of chemical kinetics, hydrodynamics, and thermodynamics of combustion. This distribution of space is, no doubt, indicative of the present state of our technical developments, which may involve more art than science in some instances. It is gratifying to note that adequate space has been allotted to recent basic studies on flame propagation, including the work by Hirschfelder and Curtiss on one-dimensional flame propagation, and Markstein's work on cellular flames. The discussion of combustion spectroscopy could bear

considerable expansion in view of its obvious importance for the ultimate elucidation of the burning mechanism.

S. S. Penner, USA

2493. **Popoff, K., On the thermodynamics of irreversible processes** (in French), *ZAMP* **3**, 1, 42–51, Jan. 1952.

The entropy deviation from an arbitrarily chosen initial state is represented by the first two terms of a power series expansion about the initial state. It is assumed that the partial derivative of the entropy deviation with respect to x_k (x_k is the deviation of a state variable presumably from the initial state rather than from equilibrium) equals d^2x_k/dt^2. Author shows for the case of two state variables $x_1 = x$, $x_2 = y$ that $dx/dt = L_{11}(d^2x/dt^2) + L_{12}(d^2y/dt^2)$, $dy/dt = L_{21}(d^2x/dt^2) + L_{22}(d^2y/dt^2)$ with $L_{12} = L_{21}$. This last expression bears a formal resemblance to the Onsager reciprocal relations in the thermodynamics of irreversible processes.

S. S. Penner, USA

2494. **Prigogine, I., and Mathot, V., Application of the cell method to the statistical thermodynamics of solutions,** *J. chem. Phys.* **20**, 1, 49–57, Jan. 1952.

The cell method for pure liquids in the form used by Lennard-Jones and Devonshire is extended to solutions. It is assumed that (a) the constituents are spherical in shape, with an isotropic field of force; (b) the distance of the maximum interaction for *AA*, *BB*, and *AB* pairs is about the same; and (c) there is random mixing.

For the mean field in the "cage," the complete 6–12 law, the harmonic oscillator, and the smoothed potential model have been studied. The smoothed potential model (potential curve with vertical walls and flat bottom, both depth and width depending on concentration) fits the liquid state best. For this model, one obtains important corrections on both the heat of mixing and the excess entropy to the classical, strictly regular solutions. These corrections are related to the volume changes on mixing resulting from the changes of interactions.

According to the value of $\epsilon_{AB}{}^*$, the excess properties such as volume, entropy, heat of mixing, and free energy present a large variety of shapes, including dissymmetry and inversions which correspond rather nicely to experimental evidence.

This model permits discussion of the severe limitations of Longuet-Higgins's recent theory of conformal solutions.

From authors' summary by S. S. Penner, USA

2495. **Denbigh, K. G., and Raumann, Gertrud, The thermo-osmosis of gases through a membrane. I. Theoretical,** *Proc. roy. Soc. Lond.* (*A*) **210**, 1102, 377–387, Jan. 1952.

When a temperature difference is set up between the two sides of a permeable membrane, migration of fluids through the membrane is induced. A reproducible pressure difference between the two film-fluid interfaces is related to the interface temperatures by an energy term analogous to but different in magnitude from the heat of solution of the fluid in the material of the film. Diffusion is an activated process; a specified amount of drifting fluid carries a proportional amount of heat with it, independent of ordinary heat conduction. The above energy term is the sum of this transported heat and the heat of solution. This explains the observation that a given temperature drop across a given film may induce different fluid materials to diffuse in opposite directions.

Jack D. Bush, USA

2496. **Denbigh, K. G., and Raumann, Gertrud, The thermo-osmosis of gases through a membrane. II. Experimental,** *Proc. roy. Soc. Lond.* (*A*) **210**, 1103, 518–533, Jan. 1952.

The basic theory of thermoosmosis of a gas through a membrane was discussed previously (see preceding review). Present article describes experimental apparatus and technique, with limitations thereof. Data are quoted for thermal diffusion of hydrogen, nitrogen, carbon dioxide, and water through membranes of different grades of rubber. In contrast to the other gases, hydrogen diffuses toward the low-temperature side of the membrane. Incidentally, the permeability of a membrane from temperature-induced diffusion is of the same order of magnitude as that from pressure-induced isothermal diffusion.

Authors point out that study of thermoosmosis may permit evaluation of intermolecular forces in condensed phases if an adequate theoretical explanation of the observed heat effects is established.

Jack D. Bush, USA

Heat and Mass Transfer

(See also Revs. 2244, 2249, 2490)

2497. **Kataoka, H., Heat conduction in a finite rod due to a given flow of heat, Parts I and II** (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17**, 62, 98–102, 102–109, Nov. 1951.

Author treats the one-dimensional heat conduction in a finite rod, when heat is supplied from a source attached to one end of the rod and is radiated not only from the other end but also through an insulator on the heat source. Part I treats the case when the heat supply is constant, and part II the case when the variation with time of the heat supply is simple harmonic.

Humio Tamaki, Japan

2498. **Sugawara, S., and Michiyoshi, I., Studies of surface heat transmission by natural convection on a flat plate, 1st and 2nd Rep.** (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17**, 62, 109–114, 115–121, Nov. 1951.

Approximate calculations of the laminar free convection from a vertical plate are presented. In the first report, authors approximate the distributions of both temperature and velocity in the boundary layer by cubics with respect to the distance y from the surface, assuming that both thermal and velocity layer have equal thickness δ, and δ is determined from the energy equation. For the plate with constant surface temperature, authors derive the formula $Nu_m = 0.525Pr^{1/4}Gr^{1/4}$, which coincides with that of ten Bosch ("Die Warmeubertragung," 1936) obtained by a different method. The case when the surface temperature varies in the vertical direction is also considered. In the second report, only the temperature distribution in the layer is assumed as polynomial with respect to y/δ, the thickness δ being assumed as the same for temperature and velocity, and the velocity distribution is determined therefrom by the equation of motion. Results show that Nusselt number is proportional to nearly $^1/_3$ power of Prandtl number. Lastly, the effect of forced convection in the vertical direction is considered.

Humio Tamaki, Japan

2499. **Miles, J. W., Heat conduction along an exponential bar,** *J. appl. Phys.* **23**, 3, p. 372, Mar. 1952.

Solution of the equation $(e^{\alpha x}f_x)x_x = e^{\alpha x}f_t$, $f = f(x,t)$ subject to the conditions $f(x,0) = 0$, $f(0,t) = 1$, $f(\infty,t) = 0$. Ed.

2500. **Sugawara, S., and Sato, T., Heat transfer in laminar boundary layer of a flat plate with temperature gradient on its surface** (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17**, 62, 137–142, Nov. 1951.

Authors treat the equation of heat transfer in laminar boundary layer for the case $T_1 - T_0 = Ax^m$, where $T_1 - T_0$ is the local temperature difference between the plate and the fluid, x is the distance from the leading edge, and A, m are constants. Neg-

lecting dissipation and variation of physical constants of the fluid, authors obtain the temperature distribution by two alternative methods: by a series expansion in $(^1/_2)(u_1/\nu x)^{1/2}y$, and by approximating it by a polynomial of the fourth degree.

Humio Tamaki, Japan

2501. Andrews, F. A., Webber, R. T., and Spohr, D. A., Thermal conductivities of pure metals at low temperatures. I. Aluminum, *Phys. Rev.* (2) **84,** 5, 994–996, Dec. 1951.

The thermal conductivities of three specimens of highly pure aluminum have been measured in the temperature range 2 to 20 K. All specimens showed maximum conductivities in the temperature range 14 to 17 K. The measured values are compared with the theory of Londheimer. The number of effective conduction electrons per atom is found to be 0.061.

From authors' summary by Frank D. Grossman, USA

2502. Rasi, A., Measurement of thermal conductivity at high temperatures (in Italian), *Ric. sci.* **22,** 1, 46–55, Jan. 1952.

A short outline on measurements of thermal conductivity of refractory materials at high temperatures. Author summarizes a few available methods for calculating thermal conductivities of solids, namely, by electrical analogies, from solutions of known steady-state problems, from nonsteady-state solution of semi-infinite slab, all with simple boundary conditions easily attainable in the laboratory.

Author is mainly interested in refractory materials where physical properties are not uniquely defined due to the complexity of their internal structure. The experimental set-up consists of refractories shaped in cylindrical form where the inside and outside temperatures are maintained constant. Knowing the electrical heat input in the center core, the temperature gradient between inside and outside surfaces, he solves for the thermal conductivity $\lambda = q \ln(r_2/r_1)/2\pi l \Delta t$. The only values of thermal conductivity available in the paper are those for silico-alumina refractory at various mean temperatures $(t_1 + t_2)/2$.

S. Eskinazi, USA

2503. Cadiergues, R., Thermometer for resultant temperature (in French), *Ann. Inst. tech. Bât. Trav. publics* (*N.S.*) no. 216, 9 pp., Nov. 1951.

The Missenard "resultant thermometer" is described. It consists of a thermometer reading from 0 to 55 C (here called °M) with its bulb enclosed in a black sphere 10 cm. in diam supported about 15 cm above its base. It has a thermal lag such that the half-time is about 2 min. It shows a correlation coefficient of 0.86 with actual physiological comfort. To yield a resultant temperature of 16 M in the center of a small room with one wall at −7 C on the outside, an air temperature of 17.7 C is required with warm air heating, and 15.2 C with radiant heating.

The Missenard thermometer is also recommended for computing heat losses from houses, as yielding much greater accuracy than air temperature. Using for the temperature driving force (°M − °C outside temperature), the inside wall heat-transfer coefficient should be taken as 8.5 instead of the classical 7.0 kcal/m^2 × hr × °C (1.74 instead of 1.43 Btu/ft^2 × hr × °F).

C. F. Bonilla, USA

2504. Sugawara, S., Sato, T., and Yoshimura, S., Experimental researches on heat transfer at the surface of a flat plate in forced flow, 1st Rep. (in Japanese), *Trans. Japan Soc. mech. Engrs.* **17,** 62, 122–129, Nov. 1951.

Local heat transfer from a thin iron plate placed parallel in uniform flow is determined from the measurement of time variation of surface temperature during the process of cooling or heating of the plate by the air flow. In the present experiments, surface temperature is not uniform along the plate, and its effect on local heat transfer is considerable in the region of laminar boundary layer. As to the turbulent heat transfer, experiments for various transition Reynolds numbers are presented by changing the shape of the leading edge and introducing a turbulence screen.

Humio Tamaki, Japan

Acoustics

(See also Revs. 2281, 2428)

2505. Smits, J. M. A., and Kosten, C. W., Sound absorption by slit resonators, *Acustica* **1,** 3, 114–122, 1951.

A discussion is given of the sound absorption by a resonator formed of a grating of parallel slits in a rigid panel mounted a distance D in front of a rigid wall.

Authors summarize previous work and give upper and lower bounds for the equivalent impedance of the panel, corresponding to the assumptions of constant pressure and constant velocity in the slits, respectively. If D is the thickness of the air space, B the lattice constant of the grating, approximate results are obtained for the case $D < 0.1\lambda$, $B < 0.5\lambda$, where λ is the wave length. Results are shown graphically and are in satisfactory agreement with the authors' measurement.

Paper concludes with a discussion of the design of this type of resonator. Convenient design charts are given, the use of which is explained.

J. M. Jackson, Scotland

2506. Stevenson, A. F., Exact and approximate equations for wave propagation in acoustic horns, *J. appl. Phys.* **22,** 12, 1461–1463, Dec. 1951.

Paper gives an exact expression for the steady-state acoustic field in an arbitrary horn in terms of an infinite number of coupled modes. In order to obtain a numerical solution, it is necessary to solve the eigenvalue problem involving the two-dimensional wave equation with boundary formed by the intersection of the horn with a plane perpendicular to the horn axis. The coefficients of the series describing the field in the horn are obtained in terms of integrals of the normal functions of the two-dimensional problem and their derivatives with respect to distance along the horn axis. An infinite set of linear equations is obtained for the coefficients. By neglecting coupling between modes, solutions for individual modes are possible. In the principal-mode case, the differential equation of motion is identical with that obtained in the conventional plane-wave approximate analysis. An important practical result of the paper is that it yields information as to the accuracy of the conventional approximation for a given horn. A detailed numerical analysis by the exact, general method of this paper would appear to be too tedious except for an extremely important problem.

George W. Swenson, Jr., USA

2507. Levin, M. L., Sound scattering in a slightly inhomogeneous medium (in Russian), *Zh. tekh. Fiz.* **21,** 8, 937–939, Aug. 1951.

Author develops the equations of motion for an arbitrary inhomogeneous medium and points out an error in Rayleigh's statement of the same equations ["Theory of sound," sect. 296]. Rayleigh used a relation between excess pressure and density in the fixed system of coordinates which is valid only in moving coordinates. The scattering formula for small inhomogeneities is also developed by the author. This coincides in form with that presented by Rayleigh, but has much wider validity because

of the afore-mentioned correction. Author also notes that these derivations show that the acoustical theorem of reciprocity is valid for an arbitrary medium in mechanical equilibrium.

R. T. Beyer, USA

2508. **Motulevich, G. P., and Fabelinskiĭ, I. L., On an optical method of controlling the character of an acoustic field** (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **81**, 5, 787–790, Dec. 1951.

In order to obtain a parallel beam of sound waves, the divergent waves can be eliminated by means of an interference filter in the form of a quartz plate of thickness $t = n\Lambda/2$ (n is an integer and Λ the wave length of sound) which is practically transparent to the sound waves at normal incidence, but reflects a considerable portion of waves incident at any other angle. Interference filters can also be made of metal, glass, or other material with small coefficient of absorption; they are more efficient for higher values of n, though they must then be set up more accurately, otherwise the unidirectional quality of waves deteriorates rather than improves. The following method can be used to verify that a filter produces the desired effect: A parallel beam of light directed along the x-axis is arranged to fall in plane sound waves in the xy-plane at an angle ϕ to the y-axis, and the intensity of the first diffracted light beam can be calculated from a theoretical formula. If the sound waves are unidirectional, almost identical graphs are obtained by theory and experiment for the intensity of the diffracted beam as a function of $(\phi + {}^1/_2(\lambda/\Lambda))$ where λ is wave length of light, the maximum in each case being taken as unity. The method of determining the energy of sound waves in any direction inclined to the normal to the quartz plate is described. The frequency of the sound waves in this work was 10^7c/s.

Marie Goyer, England

2509. **Mikhailov, I. G., and Chistorazum, A. A., Velocity of ultrasonic waves in certain binary mixtures of organic liquids** (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **81**, 5, 779–782, Dec. 1951.

It is suggested that the graphs representing the variation of the velocity of sound in binary mixtures with concentration fall into three classes: (1) They show a maximum for the mixtures in which the intermolecular forces are stronger than those in the pure liquids. (2) They are linear for mixtures approximating to the ideal. (3) They show a minimum for mixtures in which the intermolecular forces are weakened. Results are given of interferometer measurements on the velocity of sound (frequency 2869 mc/s) in binary mixtures of class (3); e.g., acetone/carbon bisulphide, methanol/carbon bisulphide. Interesting results were obtained for mixtures of acetone and chloroform which show a negative deviation from Raoul's law. Since there is a strong intermolecular force between the components, according to the above theory the variation of the velocity of sound with concentration should show a maximum, but smooth variation with a minimum is obtained experimentally, and the variation of compressibility with concentration shows a maximum.

Marie Goyer, England

2510. **Hart, R. W., and Montroll, E. W., On the scattering of plane waves by soft obstacles. I. Spherical obstacles,** *J. appl. Phys.* **22**, 4, 376–386, Apr. 1951.

Approximations are obtained for the acoustic and electromagnetic wave functions and scattering cross sections of spheres with relative index of refraction m between 1 and 1.5. The not too well motivated heuristic procedure hinges on replacing the spherical wave functions in the denominators of the coefficients of the Rayleigh-Mie infinite series by their limiting asymptotic forms for argument very large compared to order; since these denominators reduce to their correct limiting form for $m = 1$, while the asymptotic form used implies that the sphere's radius is large compared to wave length ($a \gg \lambda$), authors consider their results valid for $m \approx 1$ and hope their results have wider applicability than those of earlier approximate theories (especially for larger spheres). The procedure they follow introduces certain obvious discrepancies, their approximate coefficients being valid for only a relatively small portion of the infinite range over which the summation extends; these are negated only by the restriction to "soft spheres" $m \approx 1$, which is always in view. Their results are much better than those of the approximate Rayleigh-Gans-Born theory and are, in general, better than one might expect. Tabular and graphical data are presented and compared with results obtained by numerical summation of the original series using the Bureau of Standards tables. It is shown that for $m < 1.1$, the relative error in their total cross section is less than 5% for any a/λ; for $1.1 < m < 1.5$, the error is less than 5% if $2\pi a(m - 1)/\lambda < 6$, and less than 25% for any a/λ; for all λ the results are bounded. However, their results show no polarization effects. For $m = 1$, their results reduce to those of Van der Hulst. In the specified range $m =$ 1.0–1.5, their work should facilitate scattering calculations which have hitherto required tedious numerical manipulations; this range is of interest, for example, in the scattering of light by virus proteins.

Vic Twersky, USA

2511. **Montroll, E. W., and Hart, R. W., Scattering of plane waves by soft obstacles. II. Scattering by cylinders, spheroids, and disks,** *J. appl. Phys.* **22**, 10, 1278–1289, Oct. 1951.

The internal wave function for an infinite circular cylinder is obtained approximately, essentially as in their previous paper on spheres. This is used as the excitation factor or forcing term in the integral equation for the scattered wave; i.e., essentially, the cylinder is decomposed into elementary scatterers excited by the internal wave function. The discrepancies in this procedure stem from the approximation involved in the internal field, and are essentially as for the sphere. This same internal function is then employed to obtain the scattered wave for a finite cylinder and prolate spheroid; similarly, the internal wave function for an infinite flat plate is used to obtain the wave scattered by a finite circular disk. This procedure of neglecting the "end effects" in the excitation of the component scatterers has ample precedent; reviewer, for one, has been forced to use it in his own work on multiple scattering. Although never justified analytically, its use usually yields much better agreement with experiment than does the Rayleigh-Gans-Born or single scattering theory. The various total cross sections are determined and curves presented for the cylinder and disks; some additional material on the sphere is also included. The authors display much ingenuity in the required manipulations.

Vic Twersky, USA

Ballistics, Detonics (Explosions)

(See also Revs. 2262, 2422, 2492)

2512. **White, W. C., Rinehart, J. S., and Allen, W. A., Phenomena associated with the flight of ultra-speed pellets. Part II. Spectral character of luminosity,** *J. appl. Phys.* **23**, 2, 198–201, Feb. 1952.

Second of a series [Part I, AMR **5**, Rev. 1894] describing experiments with pellets of uniform shape and size projected through air at speeds in the range 4.5–6 km/sec summarizes results and conclusions derived from seven spectrograms of luminous trails.

Trails generated by ultraspeed pellets exhibit several types of emission spectra: continuous, line, and band. Furthermore, there is a marked difference between the spectra that depends on the composition of the pellet, i.e., aluminum is characterized chiefly by AlO bands, magnesium by MgO bands and magnesium lines, magnesium-lithium by lithium lines and MgO bands with a strong continuum. From the intensities of the AlO bands, a temperature of 4100 K was calculated.

The time-resolved spectra show that the AlO persists in the excited state twice as long as the aluminum. The vaporized aluminum oxidizes to AlO leaving AlO in an excited state.

From authors' summary by Robert M. Stewart, USA

2513. Gooding, R. C., The hydromechanics of undersea warfare, "Hydrodynamics in modern Technol.," Hydrod. Lab., Mass. Inst. Technol., 119–126, 1951.

This review, in two sections, covers hydroballistics (the study of moving submerged bodies) and underwater explosions. The first section is very brief; the second is more extensive and deals with the phenomena resulting from underwater explosions and with the hydrodynamics of explosions in free water and near a target. Much of the material is available in "Underwater explosions" by Robert H. Cole [AMR **2**, Rev. 1074], but an interesting summary of theories of damage to ship structures, etc., describes material which is not easily available. The value of this paper is diminished by the fact that author does not always specify the meaning of the symbols used in the equations.

R. M. Davies, Wales

Soil Mechanics, Seepage

(See also Revs. 2342, 2489)

2514. Tschebotarioff, G. P., Soil mechanics, foundations, and earth structures, New York, McGraw-Hill Book Co., Inc., 1951, xx + 655 pp. $6.50.

Object of the book is to present basic concepts of soil mechanics and its application to engineering structures to student engineers, both in and out of college. Author begins with origin of soil, to show fundamental physical differences in the main components, sand and clay; to describe types of tests used to classify and to measure strength, deformation, and other mechanical properties of soils; and to show how an understanding of basic principles can be used in designing engineering structures.

Object is attained by an exceptional clarity of explanation and lavish use of excellent illustration. Practice problems showing method of solution and answers further aid the student in understanding the text. An outstanding feature is the thoroughness of referencing. The young student will be continually impressed by the idea that the text is just the beginning, and that there is a broad field of study and research beyond. The engineer in practice will find these references a guide to the best literature on a particular subject of interest.

Frederick J. Converse, USA

2515. Lorenz, H., On the bearing capacity of flat foundations on sandy underground (in German), *Bautechnik* **27**, 4, 105–109, Apr. 1950.

Aim of this paper is to give pressures under a loaded strip, at which a still harmless flow may occur, in case of cohesionless soils. The flow is supposed to be still harmless if the two regions which appear separately (see Fröhlich, "Druckverteilung im Baugrund," Vienna, 1934) only touch one another, and pressures corresponding to this criterion are given for various friction angles, widths of the strip and depths of footing, as well as for the cases of centrical and eccentrical load on the strip.

From author's summary by Dragoš Radenković, Yugoslavia

2516. Terzaghi, K., Permafrost, *J. Boston Soc. civ. Engrs.* **39**, 1, 1–50, Jan. 1952.

Paper deals with problems encountered when changing the geothermal equilibrium conditions in permanently frozen ground (permafrost). Such problems are of geophysical interest, and are also becoming increasingly important for engineering operations as activities of man expand in cold regions. From a practical point of view, the most important features of permafrost are connected with volume changes due to freezing and thawing, and with loss of strength due to thawing.

Study has four parts. First part is devoted to soil and ice characteristics having a direct or indirect bearing on permafrost. Second part deals briefly with effect of freezing and thawing, including frost heave and creep, etc. Third part concerns the physics of permafrost, its prerequisites, origin, aggradation and degradation, etc. In the fourth part are discussed methods of permafrost explorations for engineering purposes.

Paper is presented on broad lines. Without using methods of refined mathematics, author gives a clear picture of basic phenomena associated with permafrost.

P. Wilh. Werner, Sweden

2517. Slezkin, N. A., On differential equations of filtration (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **79**, 5, 755–758, Aug. 1951.

The equations of motion of a mixture of two substances are stated. They may apply to a liquid and a filter substance in motion. If the velocity vector of the latter is zero, the question reduces to the motion of a liquid through an ordinary filter. In this case, the equation of equilibrium of the filter substance is added. No applications are treated.

Edgar Krahn, England

2518. Terwilliger, P. L., Wilsey, L. E., Hall, H. N., Bridges, P. M., and Morse, R. A., An experimental and theoretical investigation of gravity drainage performance, *J. Petr. Technol.* **3**, (192) 11, 285–295, Nov. 1951.

It has been recognized that decrease in oil recovery occurs at production rates above the "maximum rate of gravity drainage," defined as the production rate from a 100% saturated system under a flow gradient equal to the gravity or static pressure gradient.

A method is presented by which accurate prediction of the performance of a gravity drainage system can be made. Close agreement between experimental and calculated performance shows that steady-state relative permeability and static capillary pressure data can be used to describe fluid displacement behavior. The very wide range of liquid recoveries before gas breakthrough which result from production rate variation alone demonstrates the importance of this factor in planning depletion of a gravity drainage reservoir. Calculated results are presented which show that little additional recovery can be expected from a high-pressure gravity drainage system between the times of gas breakthrough and attainment of such high gas/liquid ratios as to make further pressure maintenance impractical.

From authors' summary by John C. Geyer, USA

2519. Murayama, S., and Hasaba, S., Studies on mixing of granular material (in Japanese), *J. Japan Soc. civ. Engrs.* **36**, 10, 456–460, Oct. 1951.

Authors present the results of their experiments on the quick segregation of mixed sand particles which are poured into a horizontal rolling cylinder and their theory thereof. They

suggest that the phenomenon can be explained by the tangential oscillation of sand mass due to centrifugal force and friction and the longitudinal shearing vibration, but the mechanism of the generation of the latter is not fully explained.

T. Mogami, Japan

Geophysics, Meteorology, Oceanography

(See also Revs. 2396, 2403, 2413, 2461)

2520. **Bultot, F., On the curvature of discontinuity surfaces of the atmosphere** (in French), *Acad. roy. Belgique, Bull. Cl. Sci.* (5) **37**, 12, 1081–1092, Dec. 1951.

The general mathematical expression which author derived in a previous paper [see AMR **5**, Rev. 1908] for the curvature of a surface of discontinuity in the atmosphere is applied to the particular case of a surface of cylindrical form, one which does not change its form in one horizontal direction—a simple type of front.

J. S. Sawyer, England

2521. **Tison, L. J., Propagation of swelling waves in a sea region** (in French), *Bull. Centre Étud. Constr. génie civ. Hyd. Fluviale* **5**, 371–386, 1951.

Article contains an analysis of the cyclic variation in flow of the Sotkamo River in Finland. The effect of variations in heat at the source of the river are discussed. Cyclic constants for various stations along the river are given. The analysis is based on an equation due to De Marchi and assumes a sinusoidal variation of heat at the river source. It is not clear that the data are of statistical significance since it is not stated whether they cover a short or long period of observation.

J. M. DallaValle, USA

2522. **Kanai, K., The result of observation of wave-velocity in the ground,** *Bull. Earthq. Res. Inst., Tokyo Univ.* **29**, part 3, 503–509, Sept. 1951.

Author obtains empirical formula

$$v = 0.6 \times (1 + 718z)/(1 + 96z)$$

(v in km/sec, z in km) for variation of P wave velocity with depth in uppermost 130 meters at Hitachi Mine, Ibaraki Prefecture, Japan.

K. E. Bullen, Australia

2523. **Nadai, A., Stress and strain in the outer solid shell of the earth,** *Trans. Amer. geophys. Un.* **33**, 2, 247–276, Apr. 1952. = Alfons Leon Gedenkschrift, Verlag Allg. Bau-Z., Wien, 4–10, 1952 (in German).

The outer shell of the earth (thickness 100 km) is first regarded as an isolated elastic sphere subject to the gravitation of the moon. The shell is so thin that bending stresses may be neglected. The moon induces tides, so that the equilibrium form of the shell is a prolate spheroid with a maximum displacement of 4.55×10^3 cm. If the core of the earth is regarded as liquid, the tides in the core increase the maximum displacement by 8.40 $\times 10^4$ cm. The hydrostatic buoyancy of the liquid core reduces these displacements by a factor of 1:2430 to 36.4 cm. Regarding the core as an elastic solid instead of a liquid leaves the theory unaltered in form, changing the interpretation of the constants.

As the earth rotates, the major axis of the prolate spheroid always lies in the equatorial plane. Thus, in addition to elastic displacements of the shell oscillating with a period of half a day, there is a mean elastic stress causing displacement of matter in the shell from the poles to the equator. This is one of the forces postulated by Wegener to account for the form of the continents. The other force, causing a westward drift, is explained by Darwin's theory of tidal friction. The major axis of the prolate spheroid does not point directly at the moon, but lags to the east. The westward pull on the end of the major axis which is nearer the moon exceeds the eastward force on the end further from the moon.

The effects of the deviation of the plane of the ecliptic from the earth's equatorial plane, and of temperature gradients in the crust are examined.

F. R. N. Nabarro, England

2524. **Hinkelmann, K., Three-dimensional problems in quantitative weather forecasting** (in German), *Meteor. Rdsch.* **4**, 11/12, 210–213, Nov./Dec. 1951.

Author reviews the equations which predict the motions of the earth's atmosphere and discusses the problem of carrying out solutions with the aid of the most modern mathematical tools, including electronic and mechanical analyzers. He makes some suggestions concerning the inclusion of effects in the vertical direction. Reference is made to the work of the meteorological group at the Institute for Advanced Study in Princeton.

W. C. Johnson, Jr., USA

Lubrication; Bearings; Wear

2525. **Kragel'skiĭ, I. V., On the calculation of the intensity of wear of rubbing surfaces** (in Russian), *Zh. tekh. Fiz.* **22**, 1, 44–54, Jan. 1952.

Tests indicate that wear caused by sliding of two surfaces is effected by interaction generated in localized contact "spots." Contacting spots can be considered of two types: destructive spots and preserving spots. The several mechanisms of destruction of a wearing surface, such as scratching, gouging, mechanical lapping, etc., may all be evaluated by numerical, weight, and linear intensities. The over-all wear intensity is obtained as the sum of the individual intensities of corresponding destructions. The magnitude of deformation due to compression A_k serves as a criterion of destruction of contacting asperities.

Expressions are developed for evaluation of the intensity of wear due to abrasion; the validity of these expressions has been verified by test data obtained by various experimenters.

L. M. Tichvinsky, USA

2526. **Nickols, L. W., and Wunsch, H. L., The design characteristics of cross spring pivots,** *Machinery, Lond.* **79**, 2030, 645–651, Oct. 1951.

Experimental investigation of 90° symmetrical cross-spring pivots of sizes used in measuring instruments. This restricts the ranges of their lengths and thicknesses to values up to $^3/_4$ in. and 0.020 in., respectively. For angular rotations up to 15 to 20° or applied torques of 6 lb-in., the torque applied to the pivot is directly proportional to the angular rotation.

Eastman's simple stiffness formula requires a modification for application to spring strips of lengths up to $^3/_4$ in.

R. Schnurmann, England

2527. **Lundberg, G., and Palmgren, A., Dynamic capacity of roller bearings,** *Acta Polyt.* no. 96 = *IngenVetenskAkad. Handl.* no. 210, 32 pp., 1952.

Experiments on fatigue life of different types of roller bearings are worked out in the light of the authors' theory [AMR 2, Rev. 1472]. The test results are expressed in terms of the usual formula, in which life is put inversely proportional to some power p of bearing load. In the two limiting cases, where either point contact or line contact is occurring in *both* raceways, p equals 3 or 4, respectively. In an intermediate, more general case, point contact occurs in one raceway, while line contact occurs on the other; then, p assumes values intermediate between 3 and 4.

When bearing load is high enough, the point contact changes into line contact, so that the intermediate case degenerates into the latter of the aforementioned limiting cases and, accordingly, p reaches the value 4.

For convenience, that is, to arrive at one general formula, the same power $p = 10/3$ is assumed to be reasonably applicable to all of the aforementioned cases. Consequently, a correction factor has to be introduced to adjust the formula as well as possible, at least within the most usual range of lives, extending from 1×10^8 to 100×10^8 revs.

Further, two reduction factors are introduced. One accounts, in all types of bearings, for stress concentrations due to edge loading as well as for that due to off-center loading of the rollers. The other accounts for a certain type of stress concentration, which occurs only in thrust-loaded bearings. Both reduction factors have been statistically evaluated from the experimental values. H. Blok, Holland

2528. Klemencic, A., Calculations and design of sleeve bearings, *J. Amer. Soc. nav. Engrs.* **64,** 1, 104–120, Feb. 1952.

In this translation of a paper in the periodical of the Society of German Engineers, 1943, some figures are omitted and others redrawn. The latter are not without faults and have too small letters and numbers. On the basis of the theory for the lubrication process with floating friction, hints and conclusions are given for the design of journal bearings. This is very good and valuable information, but it must be remembered that the "facts" in the original figures are not of general validity for all cases. The following errors are to be noted in the translation: Fig. 18, the dotted lines are not correctly redrawn; Fig. 21, "h" should be "h_0," and the numbers along the base line should be divided by 1000; Fig. 22, the numbers for the bearing clearance should be pro mille, not mm. A. R. Holm, Denmark

Marine Engineering Problems

(See also Rev. 2288)

2529. Pagès, Effect of wind action on the motion of ships (in French), *Ann. Ponts Chauss.* **122,** 2, 179–203, Mar./Apr. 1952.

Author considers equilibrium and motion of ships which arise from the combined action of wind forces and of hydromechanical forces. Application is made to ships on straight course, to maneuvers like mooring, and to the performance at anchor. Considerable simplifications (formula by Joessel for the forces, no allowance for hydrodynamical masses) make model tests advisable for further investigation. H. W. Lerbs, USA

2530. Romsom, J. A., Propeller strength calculation, *Marine Engr. nav. Arch.* **75,** 900, 901; 51–54, 117–122; Feb., Mar. 1952.

See AMR **5,** Rev. 569.

2531. van Lammeren, W. P. A., Normalization of ship propellers in Nederland (in Dutch), *Schip en Werf* **19,** 7, 133–139, Mar. 1952.

2532. Lerbs, H., The present status of theoretical research on ship propellers with respect to its technical application, *David W. Taylor Mod. Basin Transl.* 243, 18 pp., Jan. 1952.

Translation from *Werft-Reederei-Hafen* **23,** 57–62, Feb. 1942.

2533. Wilson, B. W., Ship response to range action in harbor basins, *Proc. Amer. Soc. civ. Engrs.* Separ. no. 41, Nov. 1950 = *Trans. Amer. Soc. civ. Engrs.* **116,** 1129–1157, 1951.

Paper deals with the horizontal motions of moored ships in response to wave excitation. The problem was investigated analytically, experimentally with the use of models, and by observation of prototype phenomena. Good agreement between the three methods was obtained. The critical periods of longitudinal and transverse ship motion were found to be in the range from 0 to 2 minutes. The amplitude of longitudinal motion (which is chiefly responsible for line breakage) is found to be greatly dependent on initial line tension and relatively independent of ship mass. The impact forces between ship and pier due to transverse motion were found to be approximately proportional to the square of ship mass. General conclusions are reached relating to optimum mooring practice and harbor layout for minimizing such damaging ship motion.

J. H. Carr, USA

2534. Spannhake, W., Comments and calculations on the problem of the condenser scoop, *David W. Taylor Mod. Basin Rep.* 790, 21 pp., Oct. 1951.

A comprehensive analysis of the flow through marine injection scoops and overboard discharges is given. Author attempts to formulate an exact theory for the system as a whole, using the momentum principle applied to "control spaces." An approximate theory is then presented for each of the major components, viz., scoop, condenser, and overboard discharge. Using some of the available experimental data, approximate relations are formulated for the average velocity, momentum flux, and kinetic energy available in the boundary-layer flow forward of the scoop. Definitions of system and scoop efficiencies are given; hitherto unpublished data are evaluated in these terms for one model scoop test. Final equations show that the solution depends upon four hydraulic coefficients, the values of which would have to be determined by properly controlled model experiments. A concise statement of similitude conditions to be satisfied in such experiments is next presented. Author concludes that too little is known about the flow patterns at the scoop and overboard discharge to permit an accurate prediction of the performance of any particular system. The writer advocates that future model tests should include detailed measurements of the flow patterns about separately tested scoops and overboard discharges. Reviewer believes this paper to be the most complete treatment of this subject yet produced in the marine field. John P. Breslin, USA

2535. Baier, L. A., and Ormondroyd, J., Vibration at the stern of single screw vessels, *Soc. nav. Arch. mar. Engrs.* Preprint no. 1, 16 pp., 1952.

Paper describes observations and tests leading to flow-correcting stern fins for control of large and destructive (nonresonant) fantail vibration. Model tests showed a downdraft of water ahead of propeller, and confused flow over top half of propeller. It appeared that blades passing through such water conditions at top of rotation would be heavily overloaded. Horizontal stern fins, port and starboard, fitted to the model, completely cleared up flow of water to propeller and rudder. Subsequent full-scale trials of the vessel fitted with fins resulted in reduction of vibration to one fifth its previous value in deep water, and to two fifths in shoal water. This is a very important development of great interest to marine architects and engineers.

A. C. Hagg, USA